Lecture 3
Converters I – Ideal Operation

Objectives

- Natural and Forced commutation
- To distinguish between
  - Inverters and
  - Converters
    - in the converter mode and
    - inverter mode
- Converters
  - Uncontrolled,
  - Half-controlled and
  - Fully-controlled converters

  with different forms of supply, including
  - single-phase half-wave, full-wave
  - 3-phase
- To develop general equations describing many aspects of converter behaviour

Introduction

- Inverters & Converters are the circuits which exchange energy between an AC system and a DC system.

  The two main cases:
- Systems with DC supply,
  - The circuit generate an AC source with voltage and frequency defined by the design of the circuit
  - The power flows from DC to AC and the circuit that performs this function is termed an *inverter*
- Systems with AC supply,
  - There exists an AC supply with fixed voltage and frequency (such as the mains supply) and
  - the circuit transfers power between this supply and a DC device with variable DC voltage defined by the design of the circuit

  - The circuit that performs this function is termed a *converter*
- For a converter, the power flow may be from the AC supply to a DC load and this is referred to as a *converter operating in the converter mode*
- Alternatively, the power flow may be from a DC device back to the AC supply and this is referred to as a *converter operating in the inverter mode*

It is important to understand the difference between an ‘inverter’ and a ‘converter operating in the inverter mode’

Some definitions

- For low and medium power applications - MOSFETs, bipolar transistors and gate turn-off thyristors
- For high power applications, such train motor control, thyristors have to be used
- Thyristor
  - Turn-on (i.e. made conductive)
    - gate pulse
  - Turn off
    - Reduce current flowing between the cathode and anode to below holding current
    - Maintain negative voltage for at least the turn-off time

Natural & Forced Commutation

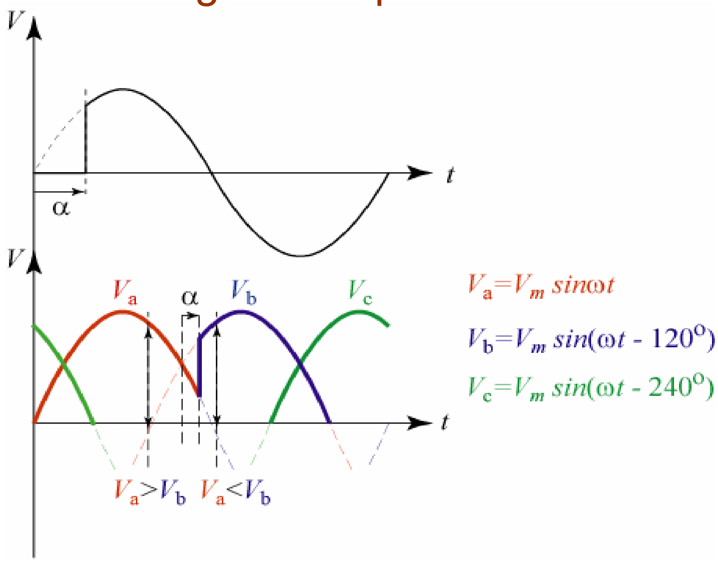
- Commutation: Process of transferring conduction from one thyristor to another.
- AC supply of converter – when a thyristor is fired the change in the voltage of the supply since the previously conducting thyristor was fired satisfies the conditions for turn-off of the previously conducting thyristor – natural commutation.
- For inverter, no AC supply – special circuitry will have to be added to satisfy the conditions for switching off the thyristors forced commutation.

Converters

- Uncontrolled, half-controlled and fully-controlled
- Uncontrolled converter or rectifier:
    - Uses diodes only – output voltage determined solely by the magnitude of the AC supply
    - Energy can only be transferred from the AC supply to the DC load.
- Half-controlled converter:
    - Uses a combination of thyristors and diodes – able to control of the DC output voltage by varying the firing angle of the thyristors
    - Energy can only be transferred from the AC supply to the DC load
    - The half-controlled converter is cheaper than a fully-controlled converter of similar rating

- Fully-controlled Converter:
    - Uses only thyristors, with control of the DC output voltage determined by the thyristor firing angles
    - Operation can either be in
        - Converter (rectifying) mode with energy transferred from the AC supply to the DC load or
        - Inverting mode with energy transferred from the DC system to the AC supply
- Firing angle
    - defines the time at which a thyristor is fired,
    - Symbol α; units radians or degrees

- Zero reference
  - Point in the cycle of the AC waveform at which a diode would conduct if the thyristor is replaced by a diode
  - Alternatively – the point in the cycle of the AC waveform when the voltage across the thyristor changes from negative to positive



- Conduction angle
  - it is the angle for which a switching device remains conducting with respect to the AC supply waveform period
- Pulse number
  - Number of discrete switching operations involving load transfer (commutation) between individual diodes or thyristors during the period covered by one cycle of the AC voltage waveform
  - The pulse number is therefore directly related to the repetition period of the DC voltage waveform (or ripple)
  - In general, the higher the pulse number, the lower the ripple amplitude

Converters with single-phase half-wave AC supply

- In a converter, diodes or switching devices, such as thyristors, are connected between the AC supply and the DC load
- AC supply voltage V can be described by:

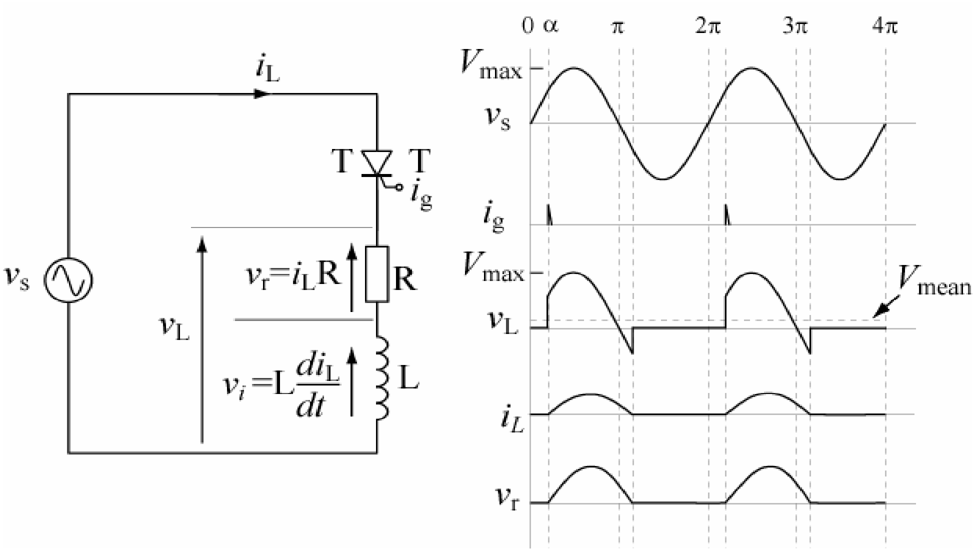
$$V = V_m \sin \omega t$$

ω is frequency of supply and V_m is peak (or maximum) value.

ωt will occur frequently – convenient to give it the symbol θ

θ is angle with the units of radians which represents normalised time scale

- AC supply voltage and the load voltage waveforms



Converters with single-phase half-wave AC supply

- When the input voltage increases from zero to positive values, the load voltage remains at zero because the thyristor is non-conducting
- Load current waveform i_L is same as thyristor current:
  - The point on the AC waveform at which the thyristor is fired by a pulse applied to its gate terminal is defined by the firing angle α
- Once the thyristor fires (at $\theta = \alpha$), the thyristor behaves like a short-circuit and the load voltage follows the supply voltage

- Current then has form of half sine-wave which falls to zero at $\theta = \alpha + \pi$; it follows that the conduction angle is π
- With inductive load, the load voltage will reverse towards the end of conduction interval
- Thyristor only stops conducting when its current goes to zero (or below holding current) and a reverse voltage is maintained across it for at least the turn-off time
- Once the current has gone to zero, and the thyristor stops conducting, the load voltage will increase to zero, maintaining a negative voltage across the thyristor as required to complete the turn-off operation

Mean Voltage

- Determining the mean load voltage
  - The mean load voltage of a converter determines the power in the DC load
  - Mean load voltage is obtained by averaging the output voltage over a whole period of the output voltage waveform
  - The average is obtained by integrating to find the area under the voltage-time curve and then dividing by the range
  - Since the period of the output voltage waveform is the same as the period of the supply waveform we should integrate over a full supply period

- Integrate from $\theta = \alpha$ to $\theta = \alpha + 2\pi$
- Output voltage is zero after the thyristor ceases conduction, i.e. after $\theta = \alpha + \pi$
- Hence we can perform the integration over the range $\theta = \alpha$ to $\theta = \alpha + \pi$

$$V_{mean} = \frac{1}{2\pi} \int_{\alpha}^{\alpha+\pi} V_m \sin\theta \, d\theta$$

- Notice that although we are integrating over the conduction angle of π, when we divide by the range, we must use the full range of 2π

  We can rewrite the equation to,

$$V_{mean} = \frac{V_m}{2\pi}\int_{\alpha}^{\alpha+\pi} \sin\theta d\theta = \frac{V_m}{2\pi}\left[-\cos\theta\right]_{\alpha}^{\alpha+\pi}$$

$$= \frac{V_m}{2\pi}\left(-\cos(\alpha+\pi) - (-\cos\alpha)\right)$$

$$= \frac{V_m}{2\pi}\left(\cos\alpha + \cos\alpha\right)$$

$$= \frac{V_m}{\pi}\cos\alpha$$

- α controls mean output voltage

- We can arbitrarily choose the starting time for the integration, but
- if a discontinuity in the output voltage waveform occurs in the middle of the integration range, then it will be necessary to carry out a separate integration for each part of the range
- This problem can be solved by choosing the integration range to begin and end with discontinuity so that only one integration is necessary.

Peak Reverse Thyristor Voltage

- Determining the peak reverse thyristor voltage
  - Can help to select thyristor can withstand this voltage
  - Voltage across thyristor equals difference between the supply voltage and load voltage
  - The peak reverse voltage occurs when the supply voltage is at its negative peak values and the output voltage is zero
  - Hence

$$\left|V_{rev-max}\right| = V_m$$

Uncontrolled Converter

- Single phase half-wave converter using a diode
- Thyristor in the converter is replaced by a diode – converter becomes uncontrolled converter
- Diode starts to conduct as soon as supply voltage becomes positive, i.e. at $\alpha = 0$
- Mean load voltage can be obtained by setting $\alpha = 0$ in controlled converter equations

$$V_{mean} = \frac{V_m}{\pi}\cos\alpha\Big|_{\alpha=0} = \frac{V_m}{\pi}$$

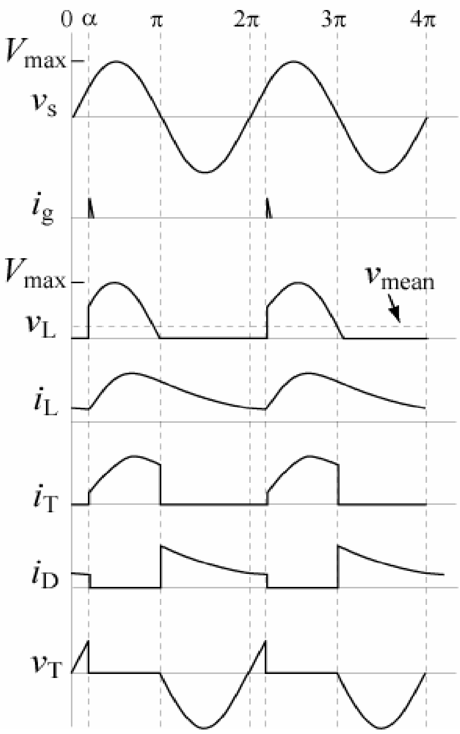
- And the peak reverse thyristor voltage

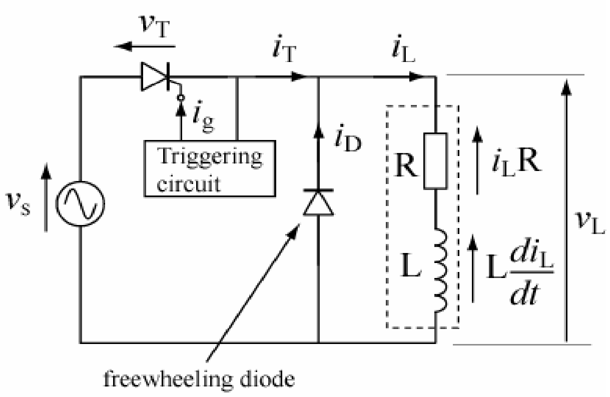
$$\left|V_{rev-max}\right| = V_m$$

- Uncontrolled converter sometimes referred to as rectifier
- The thyristor (or diode) current is equal to load current and
- thyristor has to have zero current to be able to switch off,
- Hence the load current will become zero for half of supply period
- Then we have the problem of non-constant load current, as ideal constant current is preferred.

Freewheeling diode

- Introduce a freewheeling diode in parallel with the load





- Once supply voltage has increased above zero, thyristor may be fired.
- Load voltage then follows supply voltage; diode is reverse biased, it will take no current and have no effect at this stage.
- As soon as supply and load voltage fall to zero, further change in supply voltage will forward bias diode, which will come into a conductive state and act as short circuit preventing load voltage becoming negative
- In practice, load voltage will be negative of forward voltage drop of the diode, about -0.7 V, but this will be negligible compared to typical supply voltages used in power electronics, e.g. 100V

- When diode becomes conducting, load current will switch from thyristor to diode; hence, although the thyristor current becomes zero, as required for turn-off, load current can continue to flow in diode.
- When diode is conducting, small negative voltage of -0.7 V maintained at thyristor cathode completes its turn-off.
- Kirchoff's current law at output node defines currents relationship:

$$I_L = I_T + I_D$$

- When diode is conducting, current falls slightly as current maintained by inductive part of load dissipates energy in resistive part of load.

- Rate and extent of fall depends on load time constant:

$$\tau = \frac{L}{R}$$

- Freewheeling diode considerably reduces load current variation.
- Mean load voltage
  - Average converter output voltage over conduction interval
  - Conduction is from α to π, i.e. conduction period is $\pi - \alpha$

$$V_{mean} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin\theta d\theta$$

- We can write the mean voltage as,

$$V_{mean} = \frac{V_m}{2\pi}\int_{\alpha}^{\pi} \sin\theta d\theta = \frac{V_m}{2\pi}\left[-\cos\theta\right]_{\alpha}^{\pi}$$

$$= \frac{V_m}{2\pi}\left(-\cos(\pi)-(-\cos\alpha)\right)$$

$$= \frac{V_m}{2\pi}\left(1+\cos\alpha\right)$$

  - For α = 0 and is $V_{mean} = V_m/\pi$
  - For $\alpha = \pi$, we obtain V_{mean} = 0
- Compared with fully-controlled converter, we need larger changes in α to produce given change in V_{mean}

- Peak reverse thyristor voltage

  Kirchoff's voltage law around the single loop of the converter circuit:

$$V_T = V_m - V_L$$

- Peak reverse voltage on thyristor is V_m, as before
- The diode voltage is the negative of the load voltage

- Half-wave single-phase converters have pulse number p = 1; period of output voltage ripple is equal to supply period
- Now consider converters with a pulse number of 2
- Center-tapped transformer converts single-phase supply to an effective 2-phase (bi-phase) supply
- Converters using such an arrangement are termed full-wave converters

Converters
With Single-phase Full-wave Uncontrolled

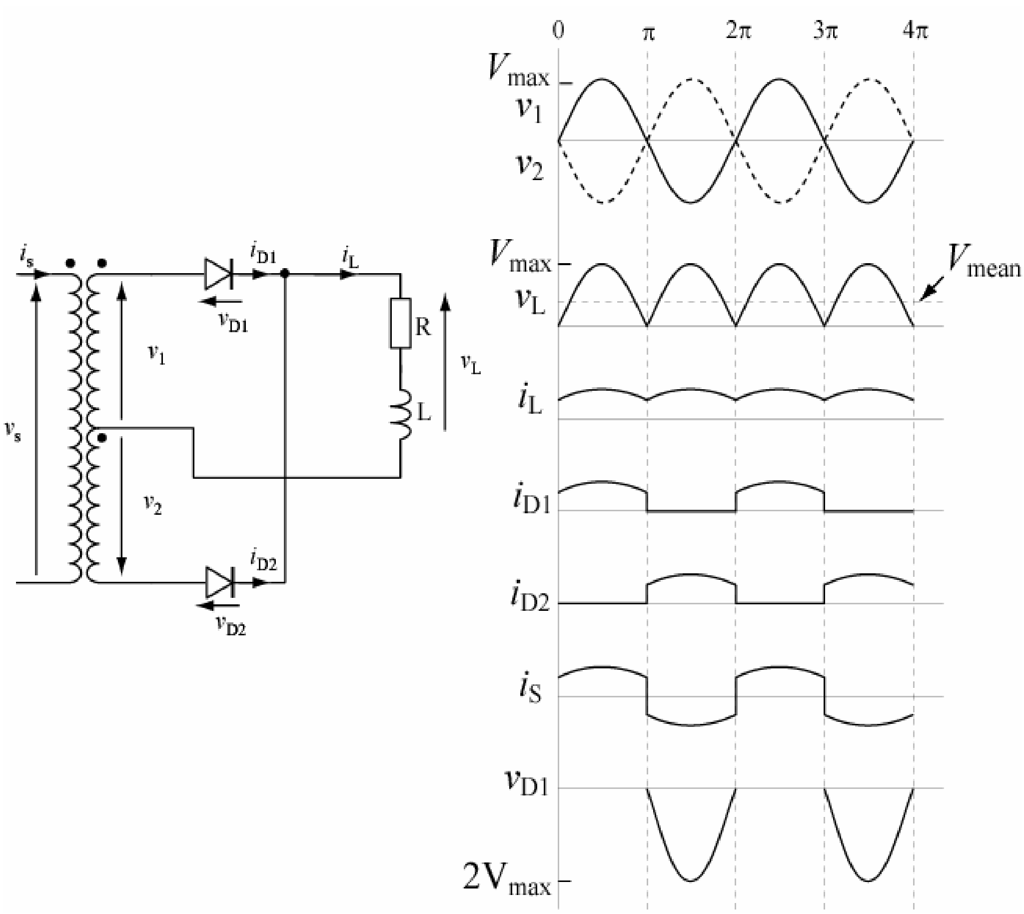
Uncontrolled converter using diodes

- Centre-tapped transformer produces anti-phase supply voltages V_1, V_2:

$$V_1 = V_m \sin\theta$$
$$V_2 = -V_m \sin\theta = V_m \sin(\theta - \pi)$$

  - V_m is amplitude of bi-phase supply voltages and $\theta = \omega t$

- Assume the load is predominantly inductive.



- Principle of operation
  - Period of output ripple is half supply period; so pulse number is 2
- Current waveforms
  - Load current is fairly constant
  - Supply current is alternating, rather than unidirectional as for half-wave circuits
- Mean load voltage
  - averaging converter output voltage over period of output voltage ripple, namely $\theta = 0$ to π;
  - Over this range, output voltage has form of positive half sine-wave:

$$V_{mean} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta d\theta$$

- We can write the mean voltage,

$$V_{mean} = \frac{V_m}{\pi}\int_0^{\pi} \sin\theta = d\theta \quad = \frac{V_m}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{V_m}{\pi}\left(-\cos(\pi) - (-\cos 0)\right)$$

$$= \frac{V_m}{\pi}\cdot 2$$

$$= \frac{2V_m}{\pi}$$

- Peak reverse diode voltage
  - Kirchoff's voltage law to loop of converter circuit

$$V_{D1} = V_1 - V_2 + V_{D2}$$

  - When D_1 is reverse biased, D_2 is forward biased and V_{D2} = 0
- Hence

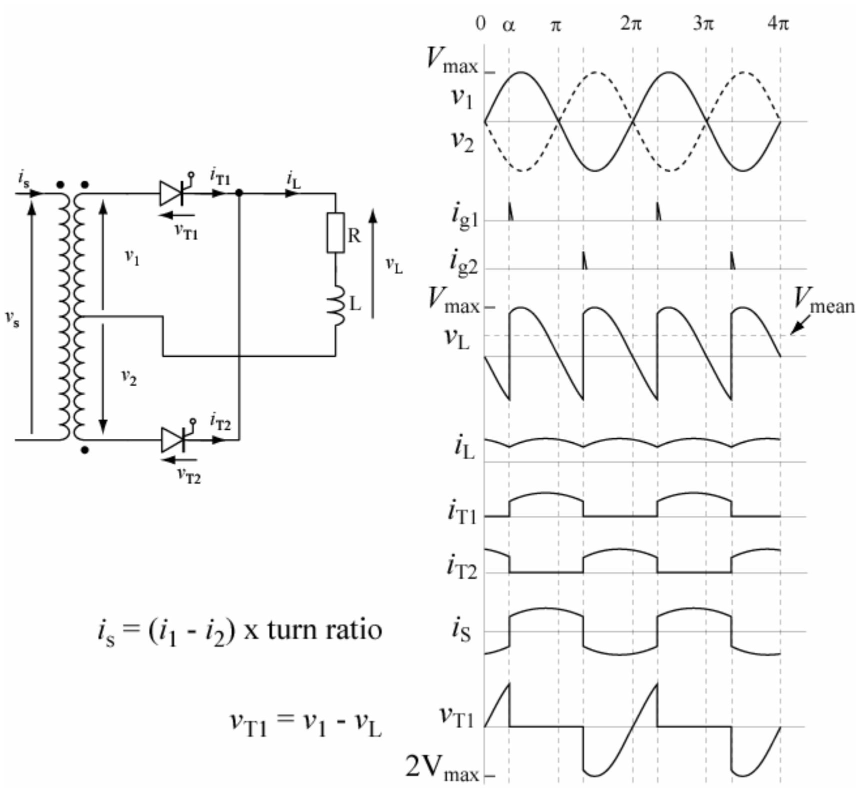
$$V_{D1} = V_1 - V_2$$

- The peak reverse voltage on the diode is $2V_m$

Converters
With Single-phase Full-wave Full-controlled

Full-wave converter using thyristors

- Assume load which is predominantly inductive
- Firing angle α for each thyristor define with respect to point on wave when diode would commence conduction
- Conduction for each thyristor is from $\theta = \alpha$ to $\theta = \alpha + \pi$; therefore conduction angle is π
- Period of output ripple is half supply period; so pulse number is 2.



- Applying Kirchoff's current law:

$$I_L = I_{T1} + I_{T2}$$

- Load current is relatively constant and supply current is alternating
- Mean load voltage
  - Average converter output voltage over its period, from θ = α to α + π:

$$V_{mean} = \frac{V_m}{\pi}\int_{\alpha}^{\alpha+\pi} \sin\theta d\theta = \frac{V_m}{\pi}\left[-\cos\theta\right]_{\alpha}^{\alpha+\pi}$$

$$= \frac{V_m}{\pi}\left(-\cos(\alpha+\pi)-(-\cos\alpha)\right)$$

$$= \frac{2V_m}{\pi}\cos\alpha$$

- Maximum mean load voltage of $2V_m/\pi$ is obtained for $\alpha = 0$
- If we set the firing angle α to 0, this is equivalent to replacing the thyristors by diodes and it can be confirmed that the mean load voltage becomes $2V_m/\pi$, which agrees with the result obtained in the previous section
- For $\alpha = \pi/2$, mean load voltage falls to zero
- Peak reverse thyristor voltage
  - Applying Kirchoff's voltage law to the converter circuit, we can write the thyristor voltage (for $T1$):

$$V_{T1} = V_1 - V_2 + V_{T2}$$

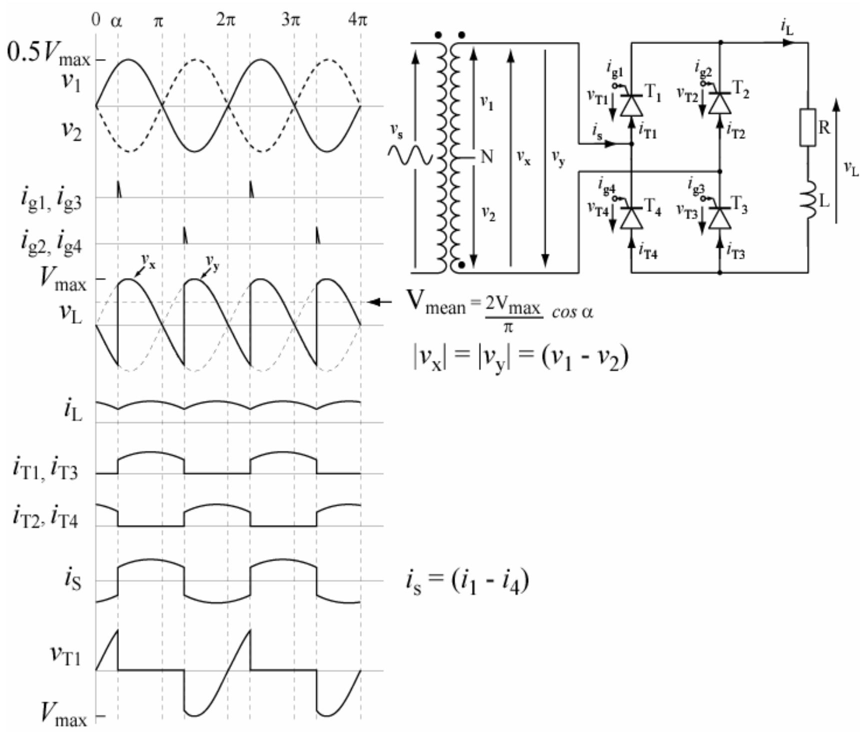
- When T_1 is reverse biased, then T_2 is forward biased and $V_{T2} = 0$

$$V_{T1} = V_1 - V_2$$

- It can be seen that the peak reverse voltage on each thyristor is $2V_m$
- Next consider another type of converter with a pulse number of 2 which we refer to as the <u>bridge converter</u>
- Replace the transformer by using <u>additional switching devices</u>

Bridge converters with single-phase AC supply

Fully-controlled bridge converter



- Circuit diagram
  - Positive (upper) terminal of load is connected to both terminals of AC supply by forward directed thyristors
  - Negative (lower) terminal of load is connected to both terminals of AC supply by reverse directed thyristors
  - Supply and load do not share common ground
- Principle of operation

  Positive half cycle of AC supply voltage is V
  - thyristors T_1 and T_3 fired together – AC supply voltage is +V

  Negative half cycle of AC supply voltage
  - thyristor T_2 and T_4 are fired – AC supply voltage is -V

- We effectively have two supply voltages (+V and –V) with load voltage tracking first one and then the other as the thyristors switch:
  - Output voltage wave form is identical to that for fullwave fully-controlled converter
- Conduction for each thyristor from θ = α to θ = α + π
- Conduction angle = π;
- Pulse number = 2

- These waveforms are identical to those for the fullwave converter
- Mean load voltage

  Averaging converter output voltage over period of output voltage, from θ = α to α + π

$$V_{mean} = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin\theta d\theta$$

  Same expression as for fullwave converter:

$$V_{mean} = \frac{2V_m}{\pi}\cos\alpha$$

  Maximum mean output voltage obtained for α = 0 and is $2V_m/\pi$; mean output voltage of zero obtained for α = π/2

- Peak reverse diode voltage

  Apply KVL to converter circuit; write thyristor voltage (for T_1):

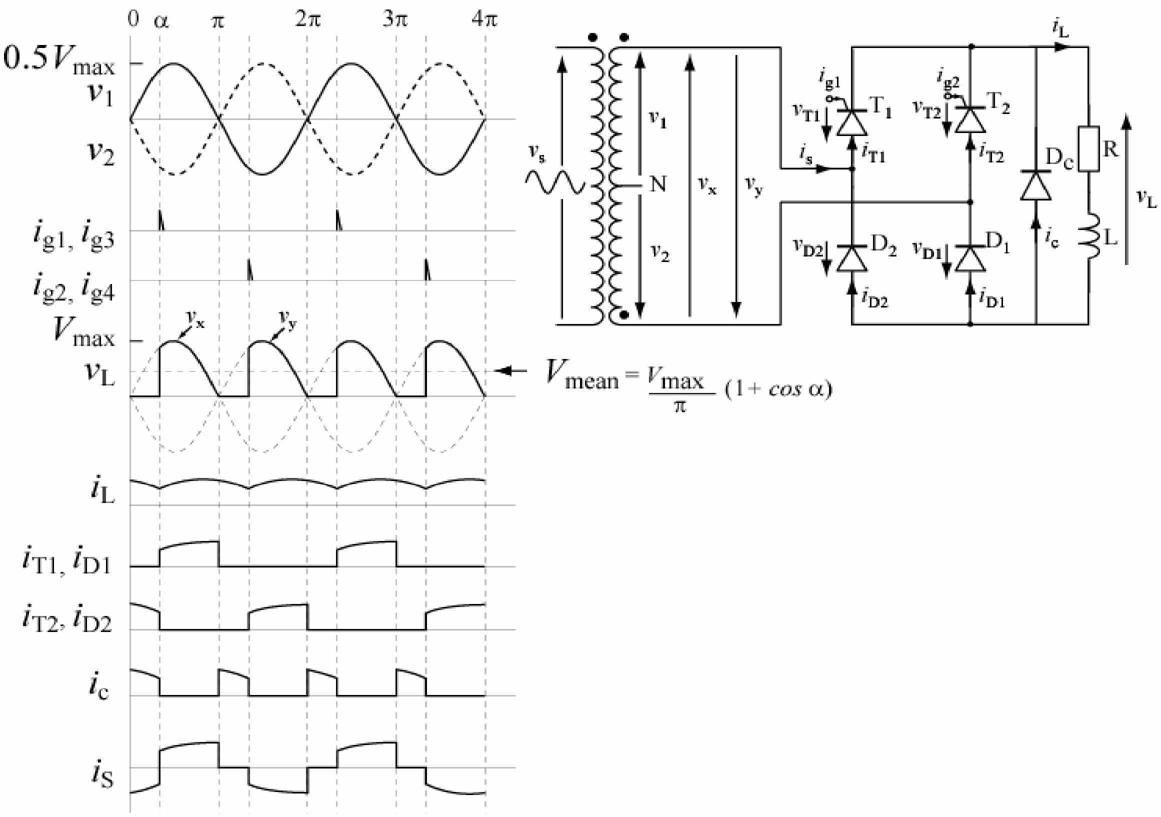
$$V_{T1} = V_1 + V_{T2}$$

  - When T_2 is forward biased,

$$V_{T2} = 0$$
$$V_{T1} = V$$

  - Maximum reverse voltage on thyristor (T_1) is V_m

Half-controlled bridge converter



- In fully-controlled bridge converter, replace thyristors T_3 and T_4 by diodes (D_1 and D_2) and introduce freewheeling diode.
- Principle of operation

  Positive half cycle of AC supply voltage V – T_1 is fired and current returns to lower terminal of AC supply via D_1

  Negative AC supply half cycle -T_2 is fired and current return is via D_2

  When supply voltage falls to zero, freewheeling diode will conduct; load current switches from thyristor to diode and thyristor is extinguished

- Pulse number is 2
- Current fall when diode conducts depends on load time constant τ = L/R
- Mean load voltage

  Thyristor conduction interval from $\theta = \alpha$ to π;
- Conduction angles, thyristors: $\pi - \alpha$; diode: α

- Average output voltage over the period of output voltage i.e. π
- Load voltage zero from $\theta = \pi$ to $\pi + \alpha$, limits of integration from α to π:

$$V_{mean} = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\theta d\theta = \frac{V_m}{\pi}[-\cos\theta]_{\alpha}^{\pi}$$

$$= \frac{V_m}{\pi}(-\cos(\pi) - (-\cos\alpha))$$

$$= \frac{V_m}{\pi}(1 + \cos\alpha)$$

- Maximum mean output voltage for $\alpha = 0$ and is $2V_m/\pi$,
- Minimum mean output voltage zero obtained for $\alpha = \pi$
- For half-controlled converter, we need larger changes in α to produce a given change in V_{mean}

- Peak reverse thyristor and diode voltages

Kirchoff's voltage law for converter circuit:

$$V_{T1} = V + V_{T2}$$

When T_2 is forward biased $V_{T2} = 0$; hence:

$$V_{T1} = V$$

The maximum reverse voltage on the reverse biased thyristor (T_1) is V_m

By a similar argument, peak reverse voltage for diodes is also V_m

Example

A highly inductive load (i.e. with constant current) is supplied from a 240V 50Hz (RMS) single-phase AC supply via a fully-controlled and a half-controlled bridge

- Compare load voltages for firing angles α of 30º and 90º

Solution

AC voltages are usually specified in RMS Volts, so it is necessary to multiply by √2 to obtain peak voltage:

- Fully-controlled bridge

$$V_{mean,30^\circ} = \frac{2V_m}{\pi}\cos\alpha = \frac{2\times 240\times\sqrt{2}}{\pi}\cos 30^\circ = 187.1V$$

$$V_{mean,90^\circ} = \frac{2\times 240\times\sqrt{2}}{\pi}\cos 90^\circ = 0V$$

- Half-controlled bridge

$$V_{mean,30^\circ} = \frac{V_m}{\pi}\left(1+\cos\alpha\right) = \frac{240\times\sqrt{2}}{\pi}\left(1+\cos 30^\circ\right) = 201.6V$$

$$V_{mean,90^\circ} = \frac{240\times\sqrt{2}}{\pi}\left(1+\cos 90^\circ\right) = 108.0V$$

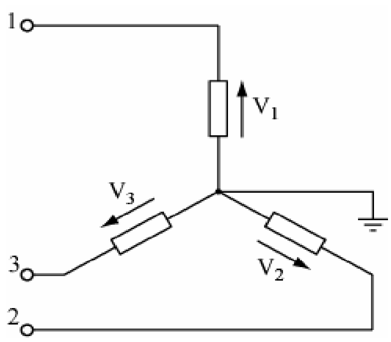
Greater sensitivity of control obtained with fully-controlled converter

Converters with 3-phase AC supply

- Single-phase converters limited to powers of few kiloWatts
- For higher power levels – converters based on 3-phase systems 3-phase supply:
  - Higher pulse numbers
  - Reduced load voltage ripple and load current ripple
- 3 lines of 3-phase supply may be connected to load in two ways:
  - Star-connection and
  - delta-connection

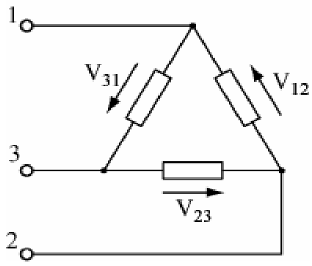
Star Connection

- In star arrangement, each load impedance is connected to supply line at one end and to common ground at other end;
- Voltages V_1, V_2 and V_3 referred to as phase voltages



Delta Connection

- In delta arrangement, each load is connected between different pairs of supply lines and none is connected to common ground;



- Delta load voltages, V_{12}, V_{23} and V_{31} referred to as <u>line voltages</u> (short for line-to-line) and are differences of phase voltages:

$$V_{12} = V_1 - V_2$$
$$V_{23} = V_2 - V_3$$
$$V_{31} = V_3 - V_1$$

Phase Voltages

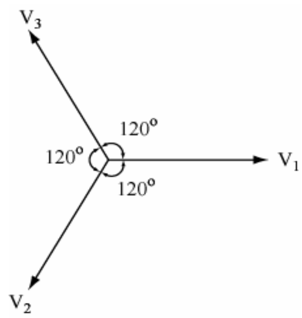
- Phase voltages

  3 phase voltages in 3-phase supply have same maximum amplitude V_m and 120º ($2\pi/3$ radians) phase differences:

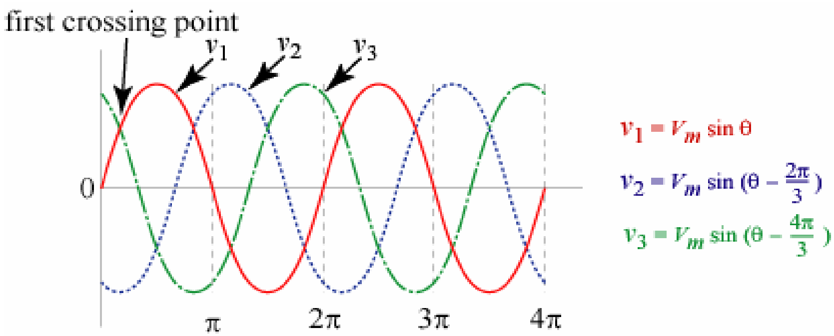
$$V_1 = V_m \sin\theta$$
$$V_2 = V_m \sin\left(\theta - \frac{2\pi}{3}\right)$$
$$V_3 = V_m \sin\left(\theta - \frac{4\pi}{3}\right)$$

where $\theta = \omega t$, V_m is peak phase voltage

- Phase voltages may be represented as vectors:



- V_1, V_2 and V_3 my be plotted against θ (or t):



- For the first crossing (of V_1 and V_3), we may write:

$$\sin\theta = \sin\left(\theta - \frac{4\pi}{3}\right) = \sin\theta\cos\frac{4\pi}{3} - \cos\theta\sin\frac{4\pi}{3}$$

$$= -\frac{1}{2}\sin\theta - \frac{-\sqrt{3}}{2}\cos\theta$$

$$\frac{3}{2}\sin\theta = \frac{\sqrt{3}}{2}\cos\theta$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6} = 30°$$

  - Crossing-points are π/6 radians or 30º after the point where one of the waveforms crosses zero

Line Voltage

- Line voltages

  Line voltages are the voltages existing between pairs of phase voltages:

$$V_{12} = V_1 - V_2 \qquad V_{23} = V_2 - V_3 \qquad V_{31} = V_3 - V_1$$

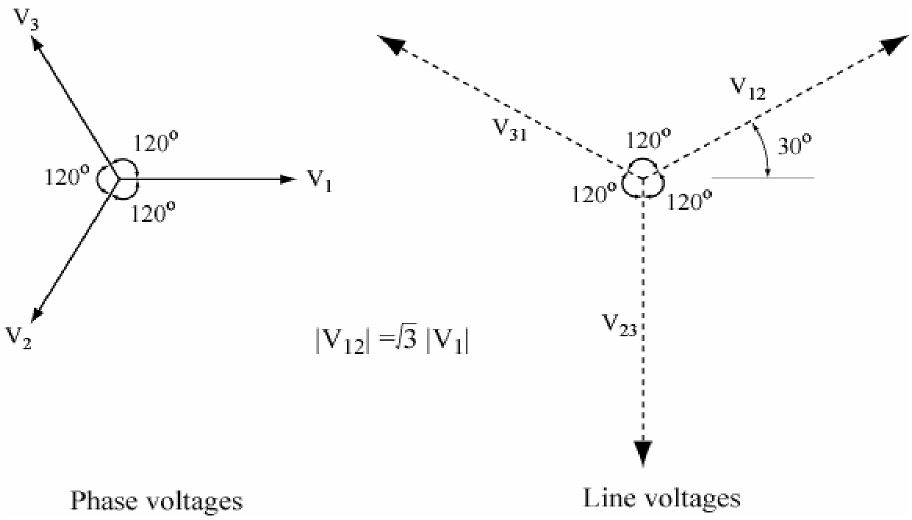
Substituting for V_1 and V_2, we obtain:

$$\begin{aligned} V_{12} &= V_1 - V_2 \\ &= V_m \sin\theta - V_m \sin\left(\theta - \frac{2\pi}{3}\right) \\ &= 2V_m \cos\left(\theta - \frac{\pi}{3}\right)\sin\frac{\pi}{3} \\ &= \sqrt{3}V_m \sin\left(\theta + \frac{\pi}{6}\right) \end{aligned}$$

- In a similar way, we obtain:

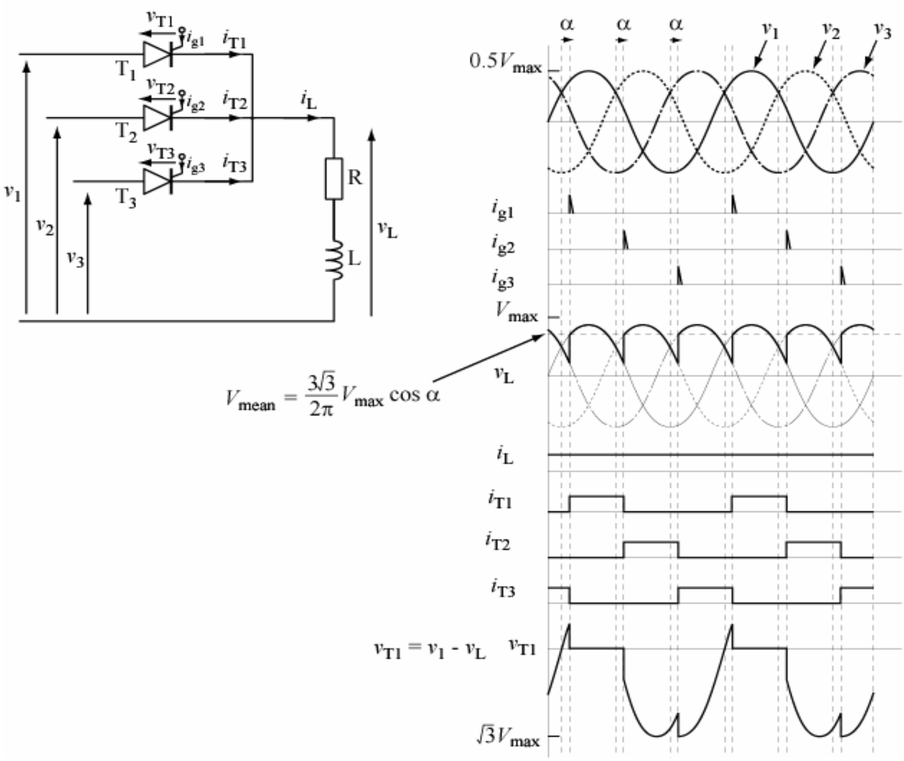
$$V_{23} = \sqrt{3}V_m \sin\left(\theta - \frac{\pi}{2}\right) \qquad V_{31} = \sqrt{3}V_m \sin\left(\theta + \frac{5\pi}{6}\right)$$

- Line voltages
  - Magnitude √3 times larger than phase voltages
  - Each differs in phase from next one by 120º (2π/3 radians)
  - Complete set rotated by 30º (π/6 radians) with respect to phase voltages:



- Important to be aware of whether specified voltage of a 3-phase supply is phase voltage or line voltage

3-phase half-wave fully-controlled converter



- Assume load is heavily inductive

- Principle of operation
  - firing angle is about 40º:
- Assume T_1 conducting and load voltage following V_1; next thyristor to be fired is T_2
  - Voltage across T_2:

$$V_{T2} = v_2 - V_L = v_2 - v_1$$

  - The earliest instant when we can fire T_2 is when V_{T2} becomes positive, i.e. when v_2 crosses v_1 (with v_2 rising and v_1 falling)
  - If T_2 was diode, this is point when it would begin conducting so it is zero reference point for firing angle a
  - Latest point T_2 may be fired is π radians later, when v_1 and v_2 cross again (with v_1 rising and v_2 falling) and voltage across T_2 changes sign again
  - Each thyristor conducts from $\theta = \alpha$ to $\alpha + 2\pi/3$;
  - Conduction angle = $2\pi/3$
  - Pulse number is 3 since period of voltage ripple is 1/3rd supply period

- Load current
  - It can be practically constant with very little ripple
  - Each thyristor conducts for 1/3rd of the supply period
  - Supply current in each phase is equal to current in the thyristor it is feeding; it follows that supply current in each phase is unidirectional and flows also for 1/3rd of supply period
  - In practice, it is desirable that supply currents are truly alternating (bi-directional); ways of achieving this will be considered later

- Mean load voltage
  - Since period of output voltage is $2\pi/3$, this will be integration range

- To avoid multiple integrations, we integrate from discontinuity to discontinuity
- We can write this

$$V_{mean} = \frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\theta d\theta$$

$$= \frac{3V_m}{2\pi} \left[-\cos\theta\right]_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}$$

$$= \frac{3V_m}{2\pi} \left(-\cos\left(\frac{5\pi}{6}+\alpha\right) - -\cos\left(\frac{\pi}{6}+\alpha\right)\right)$$

$$= \frac{3\sqrt{3}V_m}{2\pi} \cos\alpha$$

- Maximum mean output voltage obtained for $\alpha = 0$ and is $3\sqrt{3}V_m/(2\pi)$
- For $\alpha = \pi/2$, mean load voltage falls to zero

- Peak reverse thyristor voltage
  - Apply Kirchoff's voltage law to converter circuit:

    Thyristor T_1 voltage:

$$V_{T1} = V_1 - V_L$$

  - When T_1 is non-conducting, T_2 first conducts and then T_3 conducts
  - When T_2 conducts: $V_L = V_2$ and $V_{T1} = V_1 - V_2 = V_{12}$
  - When T_3 conducts: $V_L = V_3$ and $V_{T1} = V_1 - V_3 = V_{13}$
  - Hence, voltage across T_1 is equal to two line voltages, V_{12} and V_{13}
  - Maximum reverse voltage therefore = maximum line voltage = $-\sqrt{3}V_m$

- Device Ratings

  Power semiconductor devices limited by ratings – define operating boundaries within which device guaranteed to operate safely and reliably

  - Peak, average and RMS currents
  - Peak forward and reverse voltages
  - Rates of change of device current and voltage
  - Device junction temperatures

  We now determine RMS values of thyristor currents

- RMS values of voltages and currents
  - Constant DC voltage and current in a load:

- Power dissipation:

$$P = V_{dc} I_{dc} = \frac{V_{dc}^2}{R} = I_{dc}^2 R$$

- With time-varying voltage, and hence the current, power will depend on waveforms

- RMS values help us calculate power of AC signals when signals are periodic
  - RMS voltages and currents useful for specifying and determining temperature rise of switching devices such as thyristors and diodes
  - For the circuit with AC signals, instantaneous load power:

$$P(t) = \frac{V_{ac}(t)^2}{R}$$

- Heating effect depends on average power over one period
- Integrate instantaneous power over the period and divide by period T:

$$P_{av} = \frac{1}{T} \int_{t_1}^{t_1+T} \frac{V_{ac}(t)^2}{R} dt$$

- Work with angle θ rather than t, where $\theta = \omega_0 t$, with ω_0 fundamental frequency:

$$t = \frac{\theta}{\omega_0} = \frac{\theta}{2\pi f_0} = \frac{\theta T}{2\pi}$$

$$dt = \frac{T}{2\pi} d\theta$$

- Hence

$$P_{av} = \frac{1}{2\pi} \int_{\theta_1}^{\theta_1+2\pi} \frac{V_{ac}(\theta)^2}{R} d\theta$$

- We now define the RMS voltage V_{RMS} as a DC voltage which when flowing in the load generates the same power as when the AC waveform is applied; hence:

$$P_{av} = \frac{1}{2\pi R} \int_{\theta_1}^{\theta_1+2\pi} V_{ac}(\theta)^2 d\theta = \frac{V_{RMS}^2}{R} = P_{dc}$$

$$V_{RMS} = \sqrt{\frac{1}{2\pi} \int_{\theta_1}^{\theta_1+2\pi} V_{ac}(\theta)^2 d\theta}$$

- V_{ac} is first squared, then we take mean of that over one period and finally we take square-root – hence name root-mean-square

- If we had worked with the current waveform, we would have obtained:

$$I_{RMS} = \sqrt{\frac{1}{2\pi} \int_{\theta_1}^{\theta_1 + 2\pi} I_{ac}(\theta)^2 \, d\theta}$$

- For specific waveforms, ratio of RMS to peak values readily available
- For sine waves

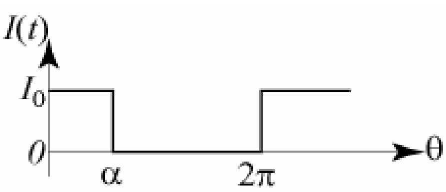
$$I_{ac}(t) = I_m \sin \omega t \quad \text{and} \quad V_{ac}(t) = V_m \sin \omega t$$

- We can easily show, using the above equation, that:

$$I_{RMS} = \frac{I_m}{\sqrt{2}} \qquad V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$P_{av} = V_{RMS} I_{RMS} = \frac{V_{RMS}^2}{R} = I_{RMS}^2 R$$

- Typical current waveform encountered in power electronics:

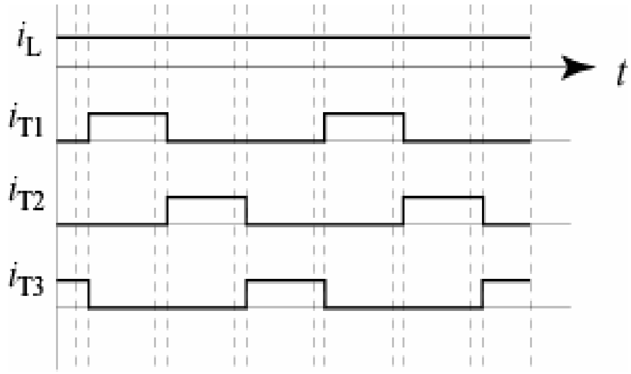


- Using above equation to calculate RMS value of current:

$$I_{RMS} = \sqrt{\frac{1}{2\pi} \int_0^{\alpha} I_0^{\,2} \, d\theta} = \sqrt{\frac{1}{2\pi} \left[I_0^{\,2} \theta \right]_0^{\alpha}} = \sqrt{\frac{1}{2\pi} I_0^{\,2} \alpha} = I_0 \sqrt{\frac{\alpha}{2\pi}}$$

- This is a very useful result that can be applied in many situations
- We now apply it to the 3-phase fully-controlled converter

- Thyristor RMS current of 3-phase half-wave fully-controlled converter



- Conduction angle is a = 2π/3 and thyristor current $I_0 = I_L$

- RMS thyristor current:

$$I_{RMS} = I_0\sqrt{\frac{\alpha}{2\pi}} = I_L\sqrt{\frac{2\pi/3}{2\pi}} = \frac{I_L}{\sqrt{3}}$$

Assuming constant load current

Example

- A 3-phase, half-wave, fully-controlled converter is connected to a 380V supply
- The load current is constant at 32A and is independent of firing angle
- Find the mean load voltage at firing angles of 0º and 45º, assuming that the thyristors have a forward voltage drop of 1.2V
- What will be the thyristor current and peak reverse voltage ratings?
- What will be the average power dissipation in each thyristor?

- *Solution*
  - When voltage of 3-phase system is specified it is usual to specify line voltage expressed as RMS quantity
  - Multiply by √2 to convert from RMS to peak and divide by √3 to convert from line voltage to phase voltage

$$V_m = \frac{380 \times \sqrt{2}}{\sqrt{3}} = 310.3V$$

  - For practical (non-ideal) thyristors and diodes, mean output voltage is reduced by device forward voltage drops
  - Mean load voltage for half-wave 3-phase fully-controlled converter less thyristor forward voltage drop:

$$V_{mean} = \frac{3\sqrt{3}}{2\pi} V_m \cos\alpha - V_T = 256.6\cos\alpha - 1.2$$

- For $\alpha = 0$:

$$V_{mean} = 256.6\cos 0 - 1.2 = 255.4V$$

- For $\alpha = 45^o$:

$$V_{mean} = 256.6\cos 45^\circ - 1.2 = 180.2V$$

- Ratings

  We have:

$$I_{rms} = \frac{I_L}{\sqrt{3}} = \frac{32}{\sqrt{3}} = 18.47A$$

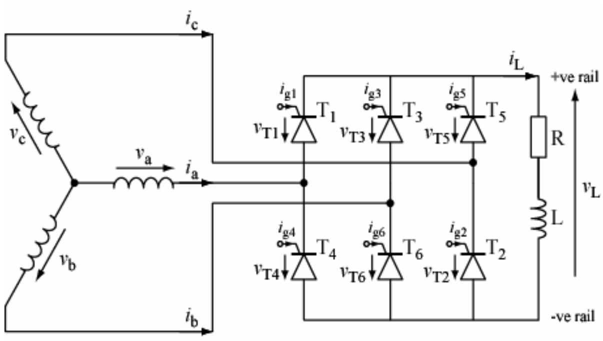
- Peak reverse voltage (PRV) is equal to peak value of AC line voltage:

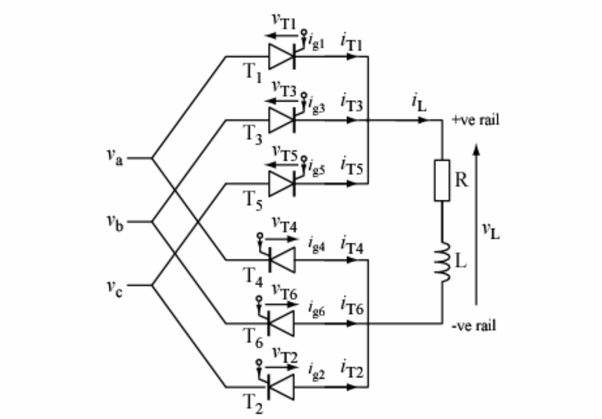
$$PRV = \sqrt{2}V_{line} = \sqrt{2} \times 380 = 537.4V$$

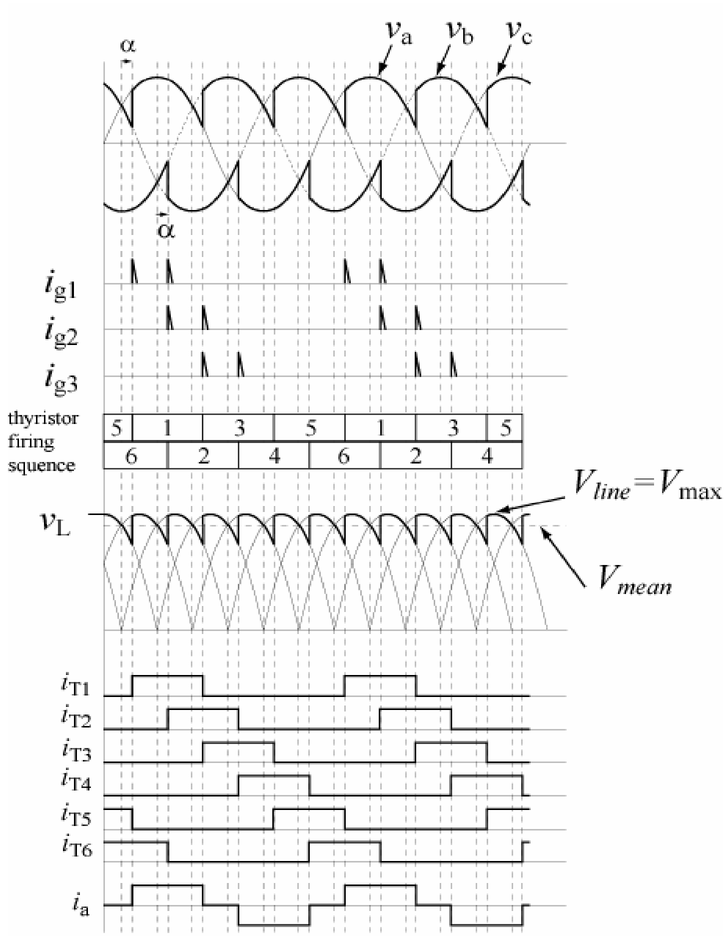
- Average power dissipation in thyristor is obtained by forming product of RMS voltage and current:

$$P_{av} = v_{t(RMS)} i_{t(RMS)}$$
$$= \frac{\hat{v}_t}{\sqrt{3}} \frac{I_L}{\sqrt{3}} = \frac{1.2 \times 32}{3} = 12.8W$$

3-phase fully-controlled bridge converter



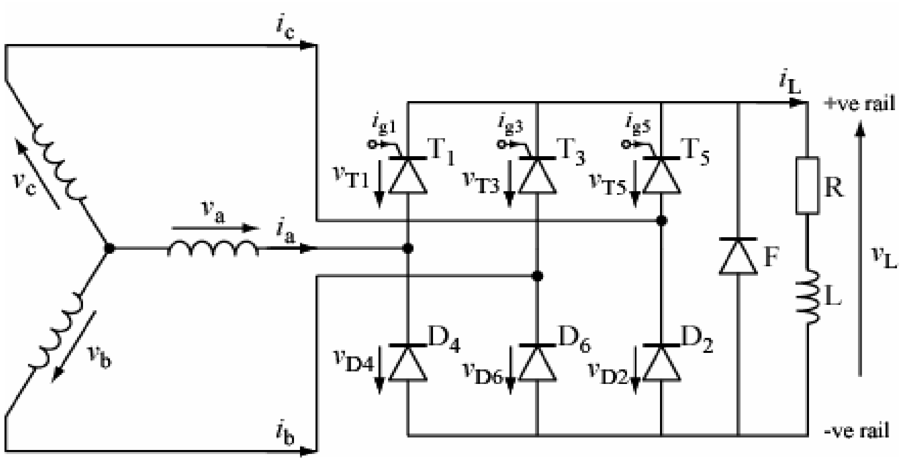


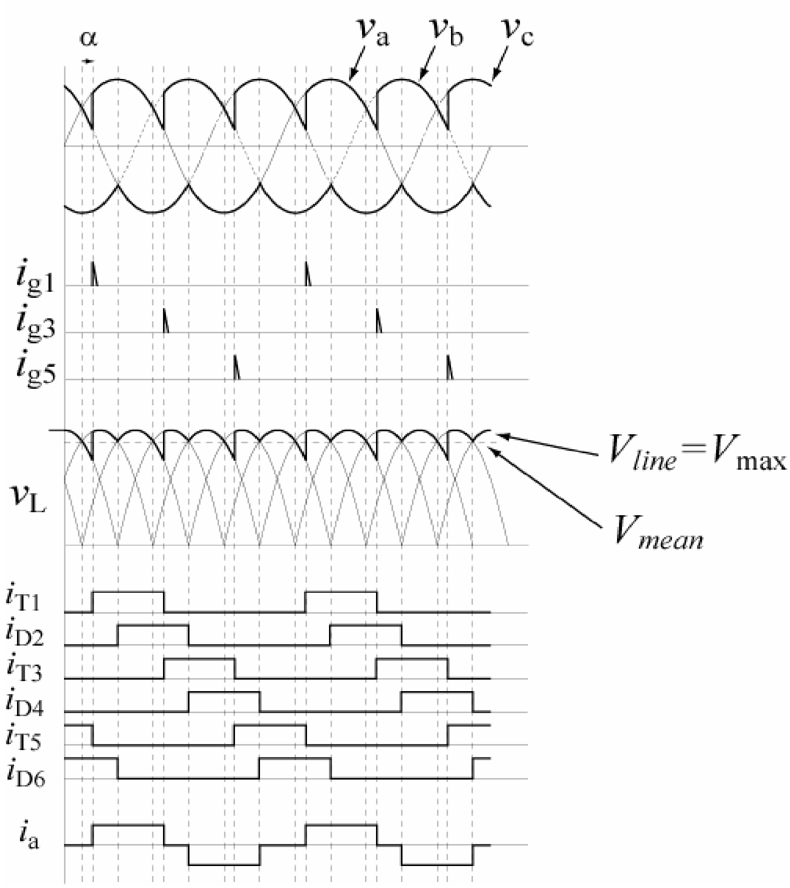


- 2 groups of 3 thyristors operate with conduction angle= 120º
- Each thyristor in lower group fired 60º after counterpart in upper group
- Load voltage = difference in output voltages of two groups
- Load voltage ripple period of 1/6th supply period
- Pulse number = 6
- 6-pulse load voltage waveform provides lower values of load voltage and current ripple than the half-wave 3-phase converter (3-pulse)
- Pairs of thyristors (one in upper group and one in lower group) my be gated simultaneously to initiate operation of converter
- Each thyristor supplied with two firing signals 60º apart; second signal has no effect on thyristor once operation has been initiated and conduction sequence established

3-phase half-controlled bridge converter

- Replace 3 lower thyristors in 3-phase fully-controlled bridge converter by diodes and add a freewheeling diode:





- Consider first operation of converter when firing angle α is $<$ 60º
- Denote positive and negative voltages on load as V+ and V-
- Load voltage given by

$$V_L = V_+ - V_-$$

- Assume value for α of about 30º
- Plot of V_+ and V_- and three phase voltages of the supply

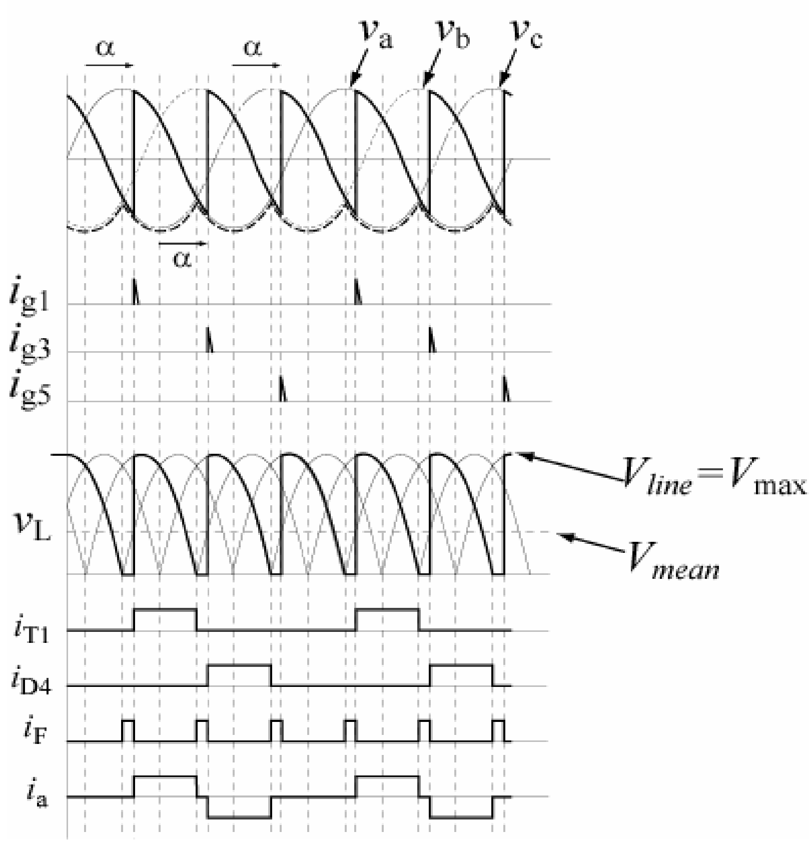
- When a thyristor fires, V_+ switches to tracking next phase voltage
- V_- switches to next phase voltage as it becomes more negative than previous one
- Load voltage V_L always equal to a line voltage
- Load voltage switches from one line voltage to another
- For $0 \leq \alpha \leq 60^o$, instantaneous load voltage is always positive and commutating diode plays no part in the operation of the converter

- Mean load voltage for $0 \leq \alpha \leq 60^o$ can be calculated by integrating appropriate sections of the line voltage waveforms:

$$V_{mean} = \frac{3}{2\pi}\left[\underbrace{\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\theta d\theta}_{\text{upper track}} + \underbrace{\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} V_m \sin\theta d\theta}_{\text{lower track}}\right] = \frac{3}{2\pi} V_m (1+\cos\alpha)$$

- V_m is peak value of supply line voltage = $\sqrt{2} \times V_{line(RMS)}$
- Maximum firing angle for this mode $0 < \alpha < 60^o$

- Plot of V+ and V- waveforms for $\alpha \approx 90^o$:
  - When firing angle reaches 60º, V_+ would become equal to V_- and hence load voltage would attempt to become negative
  - Freewheeling diode becomes conducting and takes load current during this part of cycle
  - Maximum firing angle for this mode is α = 120º



- Mean load voltage for $60 \leq \alpha \leq 120^\circ$ can be calculated by integrating appropriate section of line voltage waveform:

$$V_{mean} = \frac{3}{2\pi}\int_{\alpha}^{\pi} V_m \sin\theta d\theta$$
$$= \frac{3}{2\pi} V_m (1 + \cos\alpha)$$

Same expression as for $0 \leq \alpha \leq 60^\circ$

- For $\alpha > 60^\circ$, half-controlled converter output voltage waveform changes from 6-pulse waveform to 3-pulse waveform, losing key advantage of the 3-phase bridge converter

Summary

- Have examined simplified ideal operation of naturally commutated converters
- Considered operation of uncontrolled, fully-controlled and half-controlled converters
- Developed general equations describing converter behaviour
- Converters for single phase half-wave and full-wave supplies as well as 3-phase supplies were analysed
- For converters considered, we calculated key performance parameters including converter mean output voltage and thyristor peak reverse voltage and RMS current