# Lecture 6
# Converters IV – Design aspects

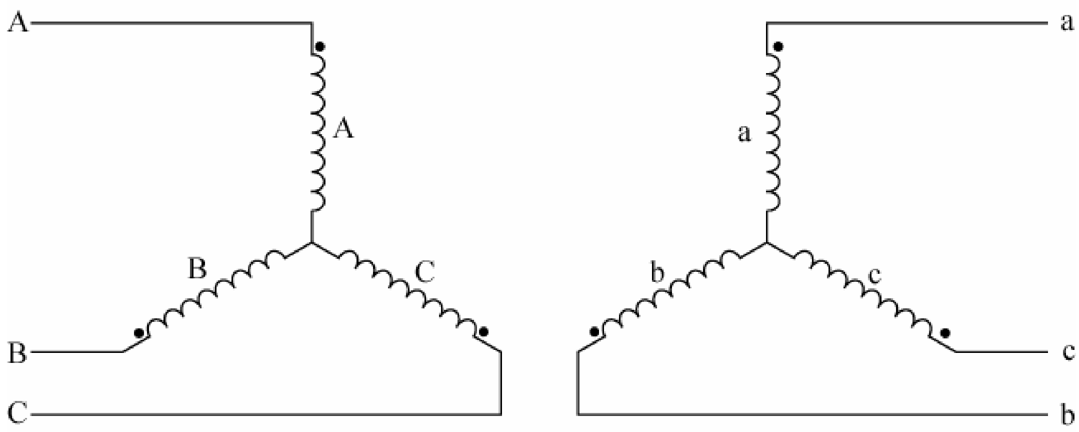


## Objectives

- We consider problem of transformer magnetising current and how this can be solved by special transformer winding schemes
- In order to reduce load voltage ripple, we consider converters with higher pulse numbers of 6 and 12; techniques include use of star and delta connected transformer windings
- We also consider miscellaneous converter topics including regulation, power factor, transformer ratings, discontinuous supply current and converters driving loads with voltage bias

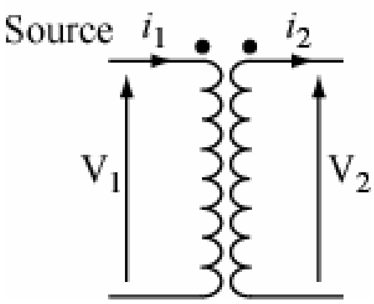
# Transformer supply and magnetising current

- In practice, converter is often connected to supply via transformer
- Star-star transformer:






- 3-phase transformer – 3 single phase transformers; primary windings A, B, C; secondary windings a, b and c
- Dot convention used with mutually coupled coils:

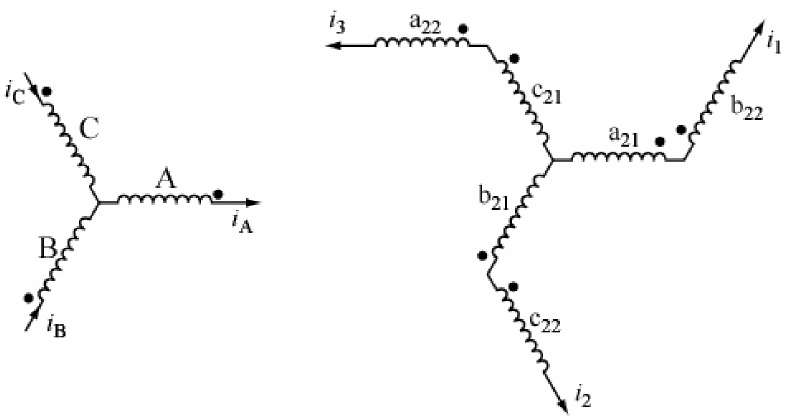



- Voltage in coupled coil has polarity indicated

- When 3-phase transformer is used to supply 3-phase converter, current in each transformer secondary is identical to current in thyristor to which it is connected, which is unidirectional
- Currents in primary windings will also be unidirectional
- DC magnetisation of transformer core – deterioration in transformer performance
- In order to prevent magnetisation of the transformer core, the interconnected star secondary winding may be used

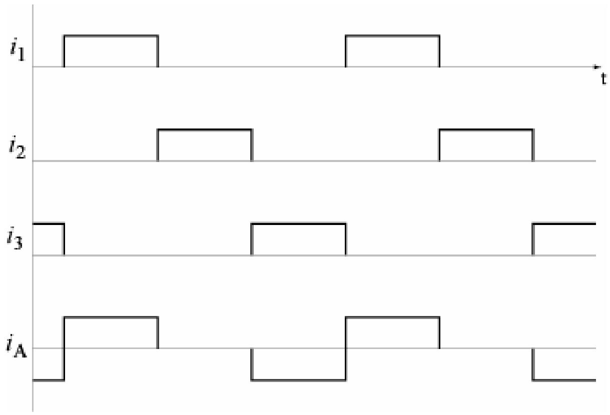


- Interconnected star secondary winding arrangement:




  - For each phase transformer, there are 3 windings; e.g. for phase A, A is primary winding and $a_{21}$ and $a_{22}$ are secondary windings
  - Diagrams show electrical interconnections and also vector relationships of winding voltages; e.g. primary windings are arranged at relative angles of 120º; secondary windings $a_{21}$ and $a_{22}$ are drawn parallel to their primary winding (A) and with correct polarity relationship

- Thyristor currents $i_1$, $i_2$ and $i_3$ and one AC supply currents $i_A$

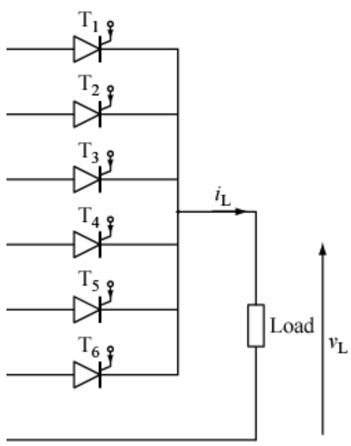



- $i_1$ causes a positive supply current and $i_3$ a negative supply current
- Interconnected star arrangement ensures bi-directional current in each transformer primary

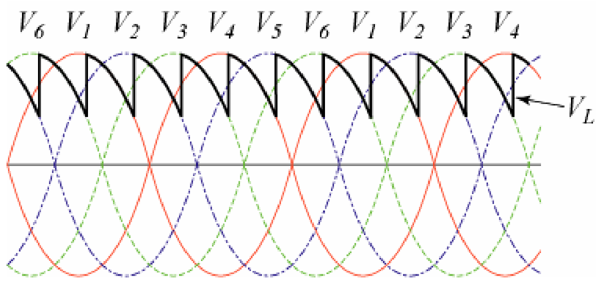


## Converters with higher pulse numbers 6-pulse systems

- Increasing pulse number reduces output voltage ripple
- We have already looked at 3-phase bridge converter (fully controlled and half-controlled) which has a 6-pulse output waveform
- Consider the 6-phase half-wave converter:



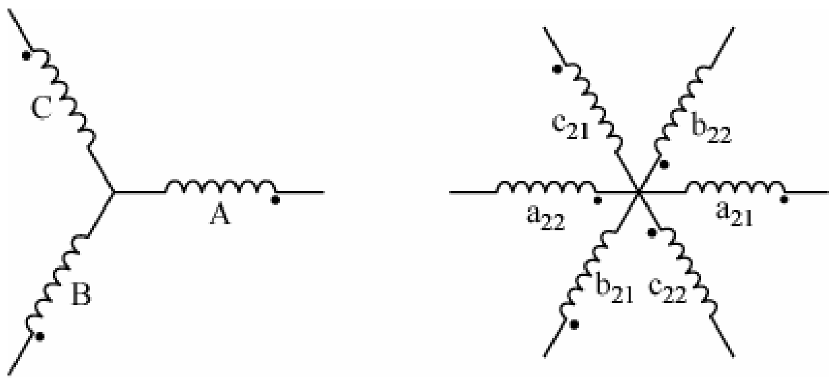
- Thyristor driven from 6-phase supply
- Supply waveforms and output voltage for $\alpha \approx 30^o$:




- Thyristors conduct for 1/6th supply period, i.e. conduction angle = 60º



- Arrangement for 6-phase supply:

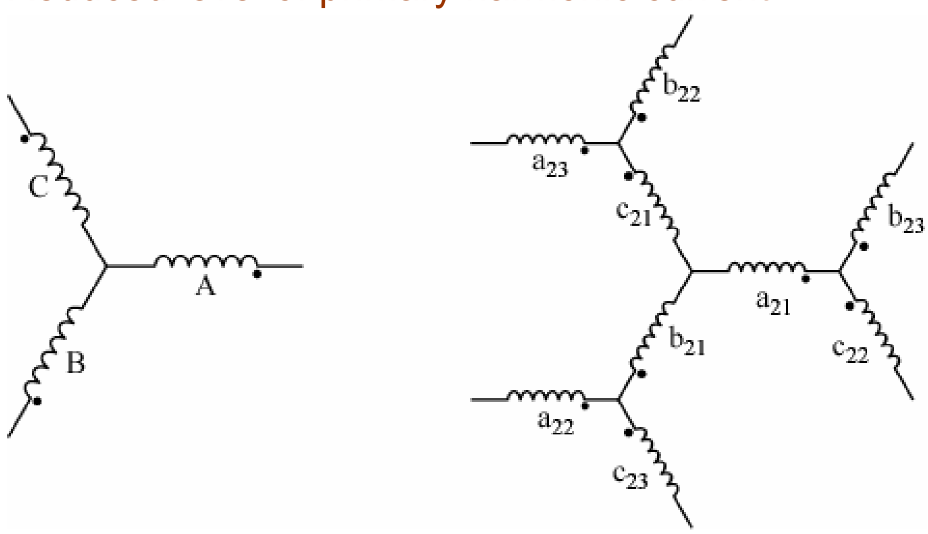



  - Converter ground return from load connected to mid-point of secondary 3-phase full-wave or 6-phase half-wave converter with 3-phase supply

- No magnetising current problems since two secondary windings for each phase make primary winding currents are bi-directional
- Current flows in each primary winding for only two 1/6ths (1/3$^{rd}$) cycle
- High levels of harmonic currents in primary circuit

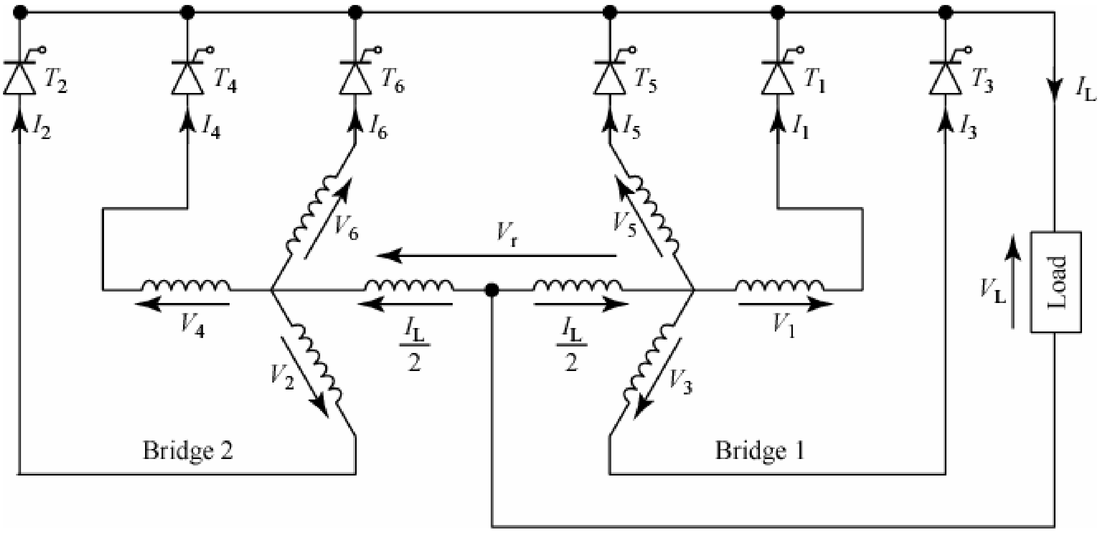


- <u>Star-fork connection</u>
  - Reduced level of primary harmonic current




  - Current flows in each phase of the primary winding for 2/3rds cycle
  - 3$^{rd}$-harmonic current in primary is zero

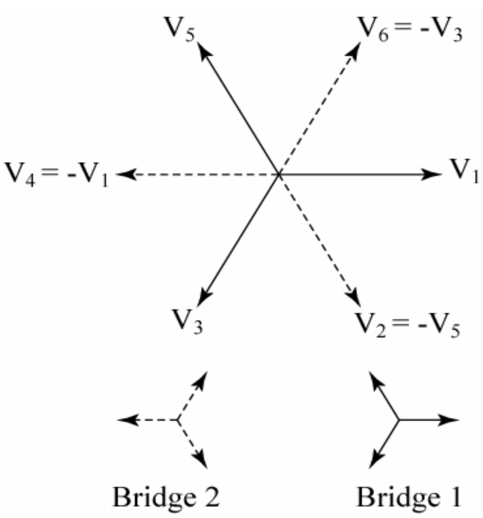
- Dual converter arrangement
  - Reduces cost associated with complex transformer windings:




  - Each group ($T_1$, $T_3$, $T_5$ and $T_2$, $T_4$, $T_6$) – 3-phase, half-wave bridge – each thyristor conducts for 120º

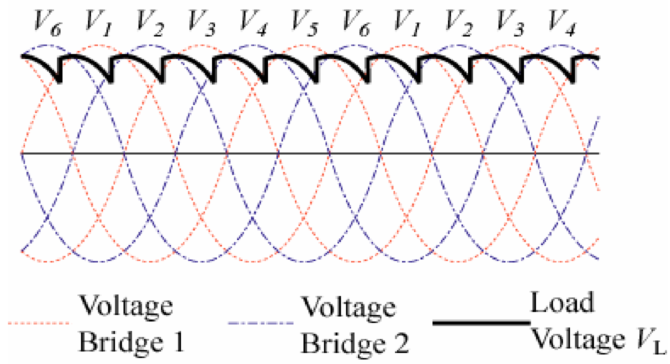


  - Secondary windings for each bridge have opposite polarity and therefore output voltage ripple transients for each converter are phase shifted with respect to each other by 60º:




  - Star points of two secondary windings of supply transformer are connected by interphase reactor

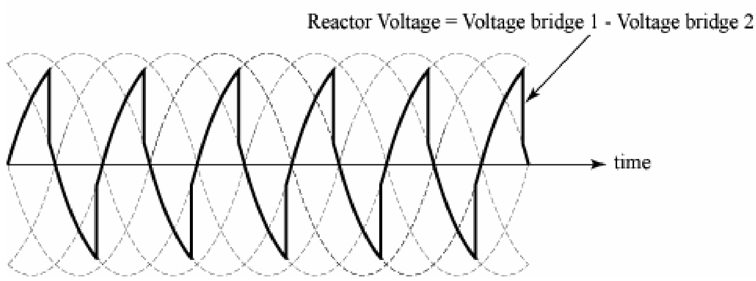
- Load voltage is mean of output voltages of individual 3-phase, half-wave groups:




- Load voltage ripple period is 1/6th of supply period $\therefore$ p = 6



- Potential difference across interphase reactor is the difference between the output voltages of individual 3-phase, half-wave converters




- Operations of interphase reactor depends on presence of circulating magnetising current which flows between two star points with return path via conducting thyristors of each group
- For magnetising current to flow, load current must be > magnetising current

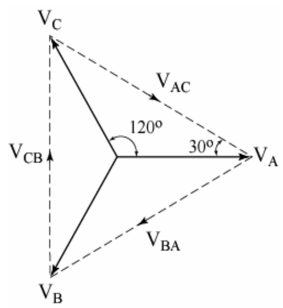
- Therefore often operated into permanently connected load
- 6-pulse systems using 6-phase half-wave converter and dual converter with interphase reactor both relatively inefficient in terms of need for special transformers and additional inductive reactors
- The 3-phase bridge configurations considered already provide 6-pulse output without need for transformers or reactors



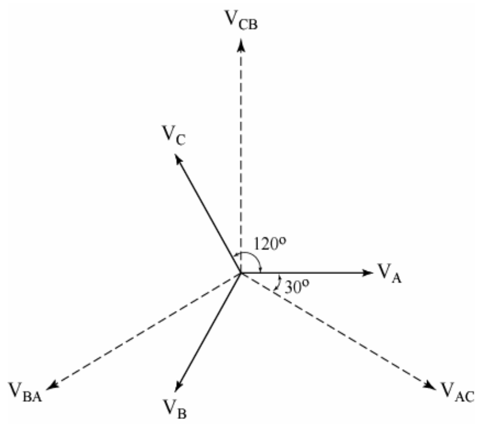
## 12-pulse systems

- Pulse numbers > 6 are needed to further reduce load voltage ripple
  - e.g. high-voltage DC (HVDC) power transmission systems
- Basic 3-phase system has phase voltages which differ in phase by 120º
- 3-phase transformer gives 6 phase voltages differing in phase by 60º

- Line voltages in 3-phase system ($V_{AC}$, $V_{BA}$ and $V_{CB}$) are differences between 3 phase voltages:

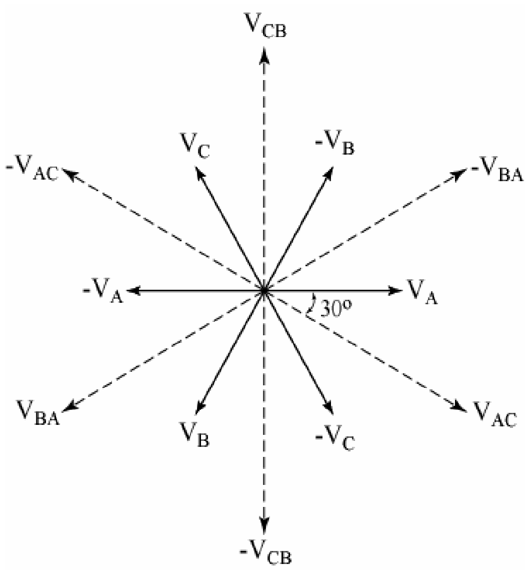



- Differ from each other by 120º; shift = 30º in relation to phase voltages:






- <u>Line and phase voltages with reversed polarity</u>:

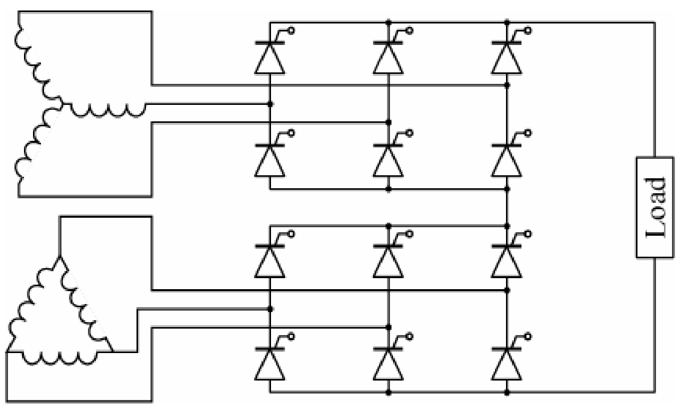



  - <u>12-phase half-wave system</u> – 30º phase difference between each phase; also described as 6-phase full-wave system

- Line voltages larger than magnitude of phase voltages by 2cos30º = √3
- Line voltages – from 3-phase supply with conventional 3-phase transformer with primary windings delta-connected
- Second transformer with star-connected primary with provide phase voltages
- Each transformer – two secondary windings for opposite polarity voltages
- Turns ratio of delta-connected transformer can be adjusted so that line voltages and phase voltages from two transformers have same magnitude

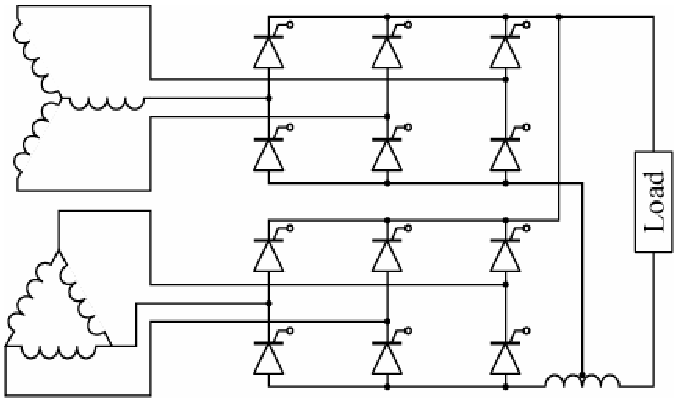


- 12 pulse converter using two 6-pulse bridges




- One bridge fed with phase voltages and one with line voltages
- Combined by connecting then in series
- Each converter is bridge converter with 60º phase shift between its upper and lower thyristor group
- Hence period of load voltage ripple is 1/12th of supply period

- Instead of combining outputs of two converters by connecting their outputs in series, they may be interconnected using a centre-tapped transformer:

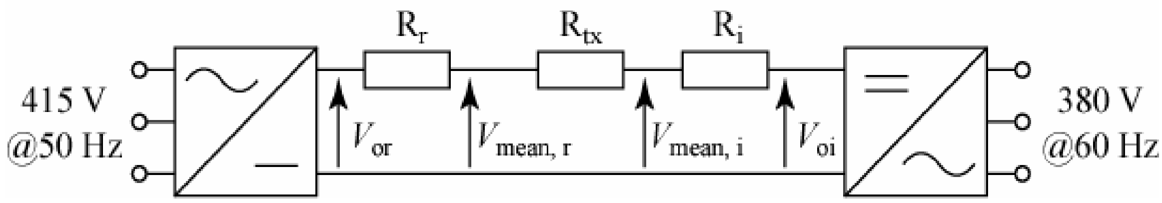





## Worked Example On High Pulse Number Converters

- Two AC supply systems are interconnected by a DC link via two 6-pulse, fully-controlled bridge converters, each consisting of two 3-phase converters with interphase reactor
  - The DC transmission line has a resistance of 0.2Ω
  - Of the two 3-phase AC systems one is 415V (line), 50 Hz and the other 380V (line), 60 Hz
  - Source inductance of the 50 Hz system is 1 mH per phase
  - Source inductance of the 60 Hz system is 1.25 mH per phase
  - If the DC link is carrying a constant DC current of 50A and delivering 15 kW into the 60 Hz system, find the firing advance angle and the firing angle of the two converters

- *Solution*
  - The equivalent circuits for the two converters, one in converting mode and one in inverting mode, can be combined with the DC link resistance to give the system equivalent circuit






- The values of $R_r$ and $R_i$ can then be calculated from the previous expressions and the given data

$$R_r = \frac{p\omega L_{(50Hz)}}{2\pi} = \frac{6\times 2\pi \times 50\times 10^{-3}}{2\pi} = 0.3\Omega$$

$$R_i = \frac{p\omega L_{(60Hz)}}{2\pi} = \frac{6\times 2\pi \times 60\times 1.25\times 10^{-3}}{2\pi} = 0.45\Omega$$

- The mean voltage for the converter in the inverter-mode can be found from the power flow and DC current:

$$V_{mean(i)} = \frac{15000}{50} = 300V$$

- The mean voltage in the absence of overlap can then be found:

$$V_{0(i)} = V_{mean(i)} - R_i I_L = 300 - 50 \times 0.45 = 277.5V$$

- The mean load voltage of the inverter-mode converter is given by

$$V_{0(i)} = \frac{p}{\pi} V_{m(i)} \sin\left(\frac{\pi}{p}\right) \cos\beta$$



$$\cos\beta = \frac{V_{0(i)}}{V_{m(i)}} \frac{\pi}{p\sin(\pi/p)} = \frac{277.5 \times \pi}{380 \times \sqrt{2} \times 6 \times \sin 30^\circ}$$

$$= 0.5408$$

$$\beta = 57.26^\circ$$

- The mean voltage for the converter in the converter-mode can be found

$$V_{mean(r)} = V_{mean(i)} + R_t I_L = 300 + 0.2 \times 50 = 310V$$

- The mean voltage in the absence of overlap can then be found:

$$V_{0(r)} = V_{mean(r)} + R_r I_L = 310 + 0.3 \times 50 = 325V$$

- The mean load voltage of the converter-mode converter is given by

$$V_{0(r)} = \frac{p}{\pi} V_{m(r)} \sin\left(\frac{\pi}{p}\right) \cos\alpha$$

$$\cos\alpha = \frac{V_{0(r)}}{V_{m(r)}} \frac{\pi}{p\sin(\pi/p)} = \frac{325 \times \pi}{\frac{415}{\sqrt{3}} \times \sqrt{2} \times 6 \times \sin 30^\circ} = 0.5799$$

$$\alpha = 54.56^\circ$$



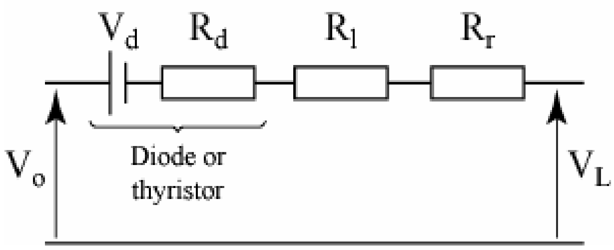
## Regulation

- Device voltage drops, device forward resistance, conductor resistance and overlap (due to AC supply inductance), cause on-load output voltage $V_{load}$ of converter to differ from ideal source voltage $V_{OC}$
- Difference expressed by regulation:

$$\text{Regulation} = \frac{V_{oc} - V_{load}}{V_{oc}} \times 100\%$$

- Voltage drop across diode or thyristor may be represented by constant voltage, combination of constant voltage and resistance

- Typical equivalent circuit:




- Parameters:
  - $V_d$ = device voltage drop
  - $R_d$ = device resistance
  - $R_l$ = lead resistance
  - $R_r$ = effective resistance due to overlap
- Precise values used must take account of firing angle



- Resistance of leads and AC source can be taken as constant
- Where bridge operation causes current to flow in two phases simultaneously, effective AC source resistance in equivalent circuit will be sum of phase resistances

## Power factor

- General expression:

$$\text{Power factor} = \frac{\text{Mean power}}{V_{RMS} I_{RMS}} = \frac{\frac{1}{T}\int_0^t vi\,dt}{V_{RMS} I_{RMS}}$$

- Converters draw non-sinusoidal current at supply frequency from AC system
- Current can be represented by fundamental component at supply frequency together with series of harmonics



- Assuming AC system voltage remains sinusoidal, there will be no power associated with current harmonics and power will be delivered by AC system at fundamental frequency only; therefore

$$\text{Mean power} = V_{1,RMS} I_{1,RMS} \cos\phi_1$$

  - $I_{1,RMS}$ is RMS amplitude of fundamental component of AC system current
  - $V_{1,RMS}$ is RMS amplitude of fundamental component of AC system voltage;
  - $\phi_1$ is phase angle between $V_{1,RMS}$ and $I_{1,RMS}$ (also referred to as phase displacement)
  - Also, we have: $V_{1,RMS} = V_{RMS}$ (for an undistorted sinewave)
  - $$I_{RMS} = \left[I_{1,RMS}^2 + I_{2,RMS}^2 + I_{3,RMS}^2 + \ldots\right]^{\frac{1}{2}}$$

- Hence, we obtain:

$$\text{Power factor} = \frac{V_{1,RMS} I_{1,RMS} \cos\phi_1}{V_{1,RMS} I_{RMS}} = \frac{I_{1,RMS} \cos\phi_1}{I_{RMS}} = \mu \cos\phi_1$$

- Where

$$\mu = I_{1,RMS} / I_{RMS}$$

  is referred to as current distortion factor
- $\cos\phi_1$ as displacement factor
- Whenever harmonic currents are present then distortion factor $\mu$ will be < 1, even if fundamental components of current and voltage are in phase ($\cos\phi_1 = 1$)
- For fully-controlled converter with constant load current, then $\phi_1$ will be equal to firing angle $\alpha$ if we ignore overlap.



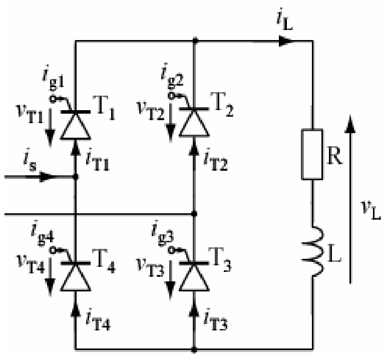
- Situation for half-controlled converter is more complicated and reference must be made to relevant AC current waveforms

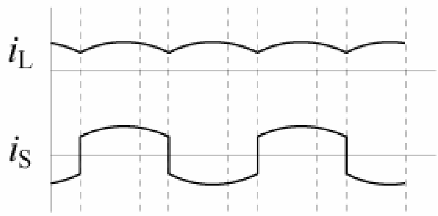
# Worked Example On Power Factor

- Part 1
  - Find the power factor for a fully-controlled, single-phase bridge converter at firing angles of 30º and 60º
  - Overlap and device forward voltage drops can be ignored and the load current is assumed to be constant



- *Solution*
- The circuit diagram of the fully-controlled, single-phase bridge converter and its load and supply current are as follows:




- Mean load voltage given by:

$$V_{mean} = \frac{2}{\pi} V_m \cos\alpha = \frac{2\sqrt{2}}{\pi} V_{RMS} \cos\alpha$$

- Assume load current constant:

- Mean load power is $V_{mean} I_L$ and power factor is:

$$PF = \frac{V_{mean} I_L}{V_{RMS} I_{RMS}} = \frac{2\sqrt{2}}{\pi} \cos\alpha$$

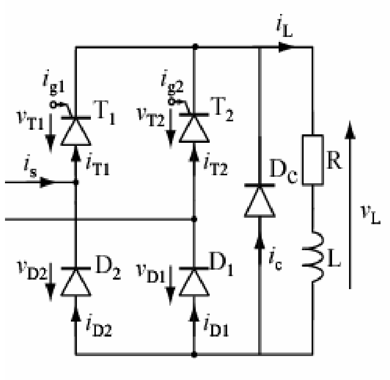
- Since the load current is assumed constant and overlap is ignored, cos $\phi_1$ = cos $\alpha$
- Also,

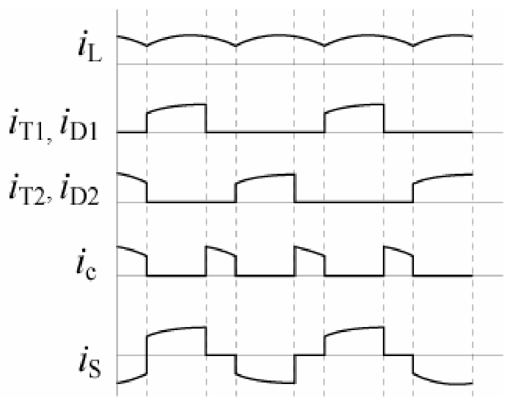
$$\mu = \frac{I_{1,RMS}}{I_{RMS}} = \frac{2\sqrt{2}}{\pi} = 0.9003$$

- Independent of firing angle
- For $\alpha$ = 30º
  Power factor = 0.7797
- For $\alpha$ = 60º
  Power factor = 0.4502



- Part 2
  - Find the power factor for a half-controlled, single-phase bridge converter for firing angles of 30 º and 60 º
  - *Solution*
    Circuit diagram of half controlled, single-phase bridge converter and its load and supply current are:



- Mean load voltage:

$$V_{mean} = \frac{1}{\pi} V_m (1 + \cos\alpha) = \frac{\sqrt{2}}{\pi} V_{RMS} (1 + \cos\alpha)$$

- Thyristor current and supply current discontinuous:
- RMS current:

$$I_{RMS} = \left[ \frac{1}{\pi} \int_{\alpha}^{\pi} I_L^2 d\theta \right]^{1/2} = I_L \left[ \frac{\pi - \alpha}{\pi} \right]^{1/2}$$

- Mean load power is then $V_{mean} I_L$ and power factor is:

$$\mathrm{PF} = \frac{\sqrt{2}}{\pi} (1 + \cos\alpha) \left( \frac{\pi}{\pi - \alpha} \right)^{1/2}$$



- By Fourier analysis, sine and cosine Fourier coefficients for the fundamental component of current are:

$$b_1 = \frac{1}{\pi} \left[ \int_{-(\pi-\alpha)}^{0} -I_L \sin\theta d\theta + \int_{\alpha}^{\pi} I_L \sin\theta d\theta \right] = \frac{2}{\pi} I_L (1 + \cos\alpha)$$

$$a_1 = \frac{1}{\pi} \left[ \int_{-(\pi-\alpha)}^{0} -I_L \cos\theta d\theta + \int_{\alpha}^{\pi} I_L \cos\theta d\theta \right] = \frac{2}{\pi} I_L \sin\alpha$$

- Amplitude of fundamental current component:

$$\hat{I} = \left( a_1^2 + b_1^2 \right)^{1/2} = \frac{2\sqrt{2}}{\pi} I_L (1 + \cos\alpha)^{1/2}$$

- RMS value of fundamental current component:

$$I_{RMS} = \frac{2}{\pi} I_L \left(1 + \cos\alpha\right)^{1/2}$$

- Hence

$$\mu = \frac{2}{\pi}\left(\frac{\pi}{\pi - \alpha}\right)^{1/2}\left(1 + \cos\alpha\right)^{1/2}$$

- From expression for power factor we obtain:

$$\cos\phi_1 = \frac{1}{\sqrt{2}}\left(1 + \cos\alpha\right)^{1/2}$$

- For $\alpha$ = 30º
  $\mu$ = 0.9526 $\cos\phi_1$ = 0.9659 Power factor = 0.9201
- For $\alpha$ = 60º
  $\mu$ = 0.8541 $\cos\phi_1$ = 0.866 Power factor = 0.7397



## Transformer rating

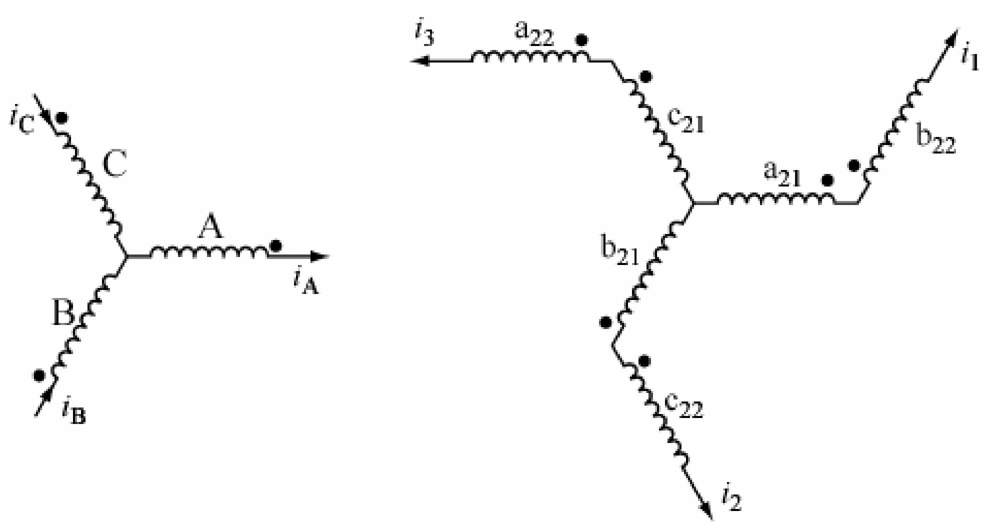
- When selecting transformers their rating under particular operating conditions must be determined
- Rating will in many cases be different for primary and secondary windings
- In normal transformer operation rating will be same for both windings
- Rating of a winding is obtained as product of RMS current through winding and RMS voltage across winding

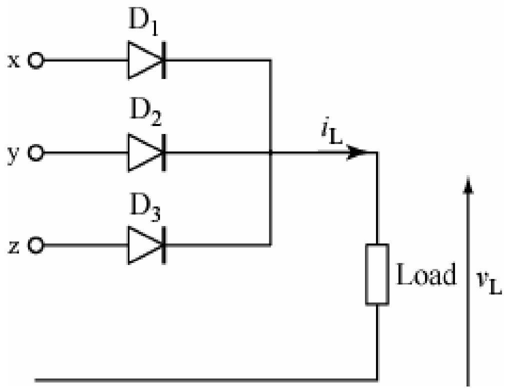
# Worked Example On Transformer Ratings

- 3-phase, half-wave, uncontrolled converter is supplying constant current of 25 A at 240 V to load
- Converter is supplied from secondary of interconnected star transformer, primary of which is connected to 3-phase, 660 V (line) supply
- Find ratings of transformer primary and secondary windings



- *Solution*
  - 3-phase uncontrolled converter + interconnected star transformer drive:






$$V_{mean} = \frac{3\sqrt{3}}{2\pi} V_m \cos\alpha \Big|_{\alpha=0} = \frac{3\sqrt{3}}{2\pi} V_m$$

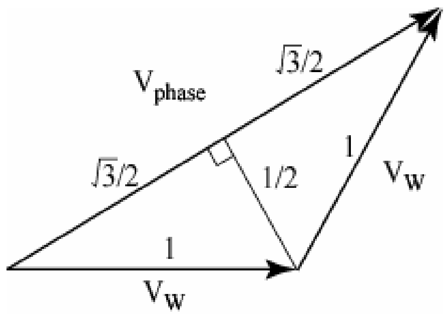
- $V_m$ is peak phase voltage of AC supply at transformer secondary



$$V_m = \frac{2\pi}{3\sqrt{3}} V_{mean} = \frac{2\pi}{3\sqrt{3}} 240 = 290.2\,V$$

$$V_{RMS} = \frac{290.2}{\sqrt{2}} = 205.2\,V$$

- In interconnected star, each secondary phase voltage is developed as vector of two secondary winding voltages $V_w$:



- It follows from geometry that $V_{phase} = \sqrt{3} V_w$
- RMS winding voltage:

$$V_{W.RMS} = \frac{V_{RMS}}{\sqrt{3}} = \frac{205.2}{\sqrt{3}} = 118.4\,V$$

- Load current $I_L$ flows for 1/3rd of cycle in each transformer secondary; RMS secondary current:

$$I_{2.RMS} = \left(\frac{1}{3} I_L^2\right)^{1/2} = \frac{I_L}{\sqrt{3}} = \frac{25}{\sqrt{3}} = 14.43\text{ A}$$

- Primary winding (phase) voltage is obtained from line voltage:

$$V_{1.RMS} = \frac{V_{line}}{\sqrt{3}} = \frac{660}{\sqrt{3}} = 381\,V$$



- Turns ratio between primary winding and associated secondary windings:

$$R = \frac{n_p}{n_s} = \frac{V_{1.RMS}}{V_{W.RMS}} = \frac{381}{118.4} = 3.218$$

- Primary current amplitude:

$$I_1 = \frac{25}{3.218} = 7.77\text{ A}$$

- Primary current flows for 2/3rds of cycle
  $\therefore$ RMS primary current:

$$I_{1.RMS} = \left(\frac{2}{3} I_L^2\right)^{1/2} = \frac{\sqrt{2}}{\sqrt{3}} I_1 = \frac{\sqrt{2}}{\sqrt{3}} 7.77 = 6.34\text{ A}$$

- Transformer ratings:
  Primary

$$P_p = 3 \times 6.34 \times 381 = 7.25 \text{ kW}$$

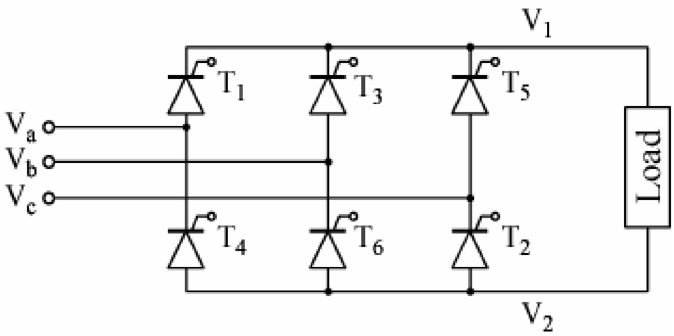
  Secondary

$$P_s = 6 \times 14.43 \times 118.4 = 10.2 \text{ kW}$$



# Converters with discontinuous current

- Where load inductance is insufficient to maintain DC current constant, output current will contain a ripple component which will appear in supply current
- e.g. 6-pulse bridge converter fed from 3-phase supply

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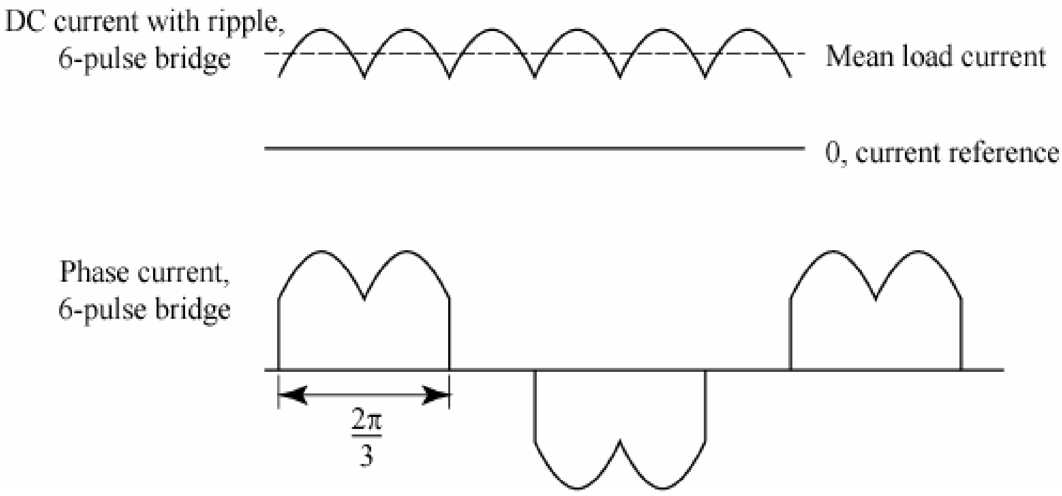
|---|---|---|---|---|---|---|
| 5 | 1 | 3 | 5 | 1 | 3 | 5 |
| 6 | 2 | 4 | 6 | 2 | 4 | 6 |

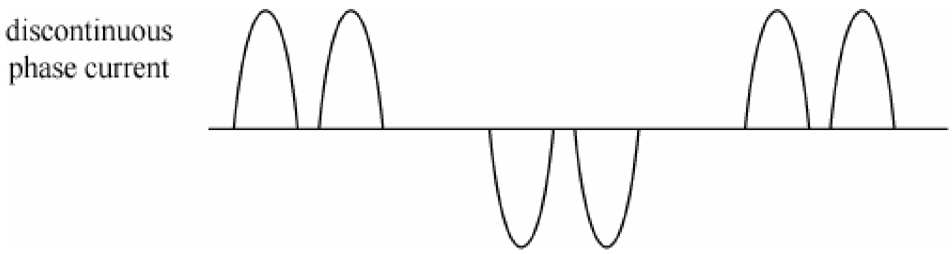
- Current flows in phase A of AC supply when $T_1$ or $T_4$ are conducting

- Load current and phase A supply current when load current has significant ripple:

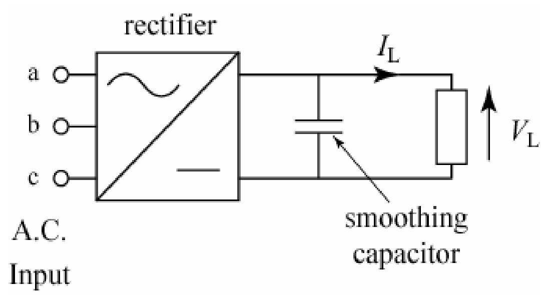




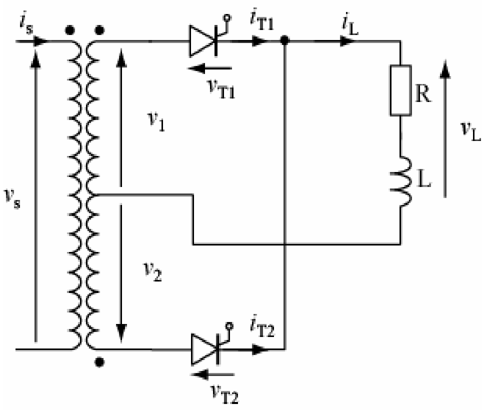

- Under light load conditions:



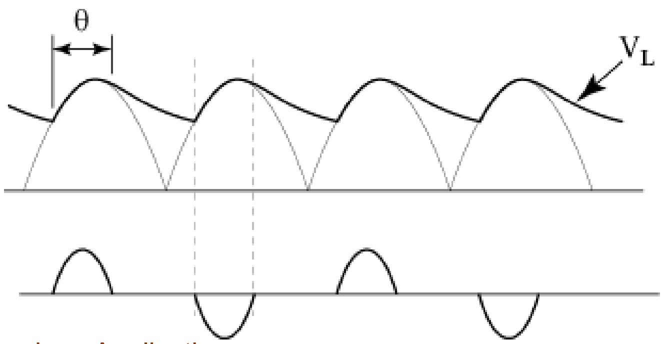

  - Amplitude of current ripple exceeds mean load current
  - Analysis of converter is much more complex in this case
  - Capacitor smoothing of an uncontrolled converter (or rectifier) can make supply current discontinuous:



- e.g. 2-phase, half-wave bridge with capacitor smoothing:




- Diodes conduct once AC voltage at anode exceeds capacitor voltage:





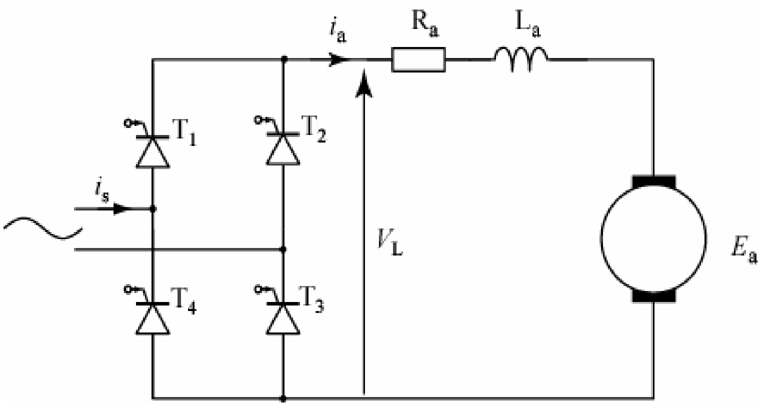

- Conduction period θ can be approximated as:

$$\theta = 2\cos^{-1}\left(\frac{V_{mean}}{V_m}\right)$$

- $V_m$ is peak value of AC supply at transformer secondary

## Converters with voltage bias

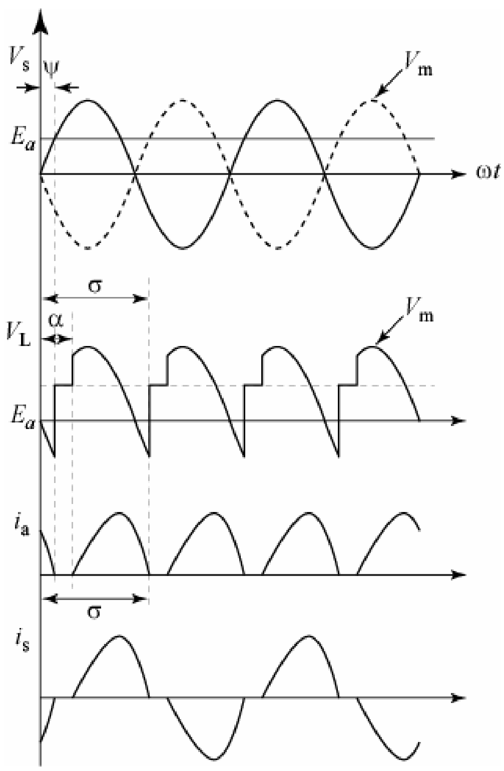
- Consider fully-controlled converter supplying DC motor load:




- Back EMF of motor appears as bias voltage $E_a$ on DC side of converter



- Supply voltage and load voltage:



- α – firing angle of the thyristors
- ψ - point-on-wave at which AC source voltage exceeds bias voltage
- σ - point-on-wave when same zero of AC supply voltage

- α, ψ and σ all measured from same zero of AC supply voltage
- Performance of converter under these conditions depends on relationship between parameters
- Thyristors cannot be fired for α < ψ
- Effective load voltage:

$$v_L = V_m \sin\theta \quad \text{for } \alpha < \theta < \sigma$$

$$v_L = E_a \quad \text{for } \sigma - \pi < \theta < \alpha$$



- Mean load voltage:

$$v_{mean} = \frac{1}{\pi}\left[\int_{\sigma-\pi}^{\alpha} E_a d\theta + \int_{\alpha}^{\sigma} V_m \sin\theta d\theta\right]$$

$$= \frac{1}{\pi}\left[V_m(\cos\alpha - \cos\sigma) + E_a(\alpha + \pi - \sigma)\right]$$

- Supply current becomes discontinuous:

# Summary

- Have concluded topic of converters by considering some advanced design aspects:
  - Problem of transformer magnetising current
  - Development of converters with higher pulse numbers of 6 and 12
  - Other important topics including power factor and regulation
- Next we consider alternative class of systems in which natural commutation can not be used and we have to provide forced commutation
- This includes important classes of circuit, such as:
  - DC choppers
  - Inverters