Lecture 9
Inverters

Objectives

- To examine operation of inverters as means of producing variable-frequency, variable voltage AC source from DC supply
- To consider inverters which produce single-phase and 3-phase AC outputs
- To consider both voltage-sourced inverters which operate from DC supply which approximates constant voltage source and current-sourced inverters which operate from DC supply which approximates constant current source
- To introduce pulse-width-modulated inverters

Introduction

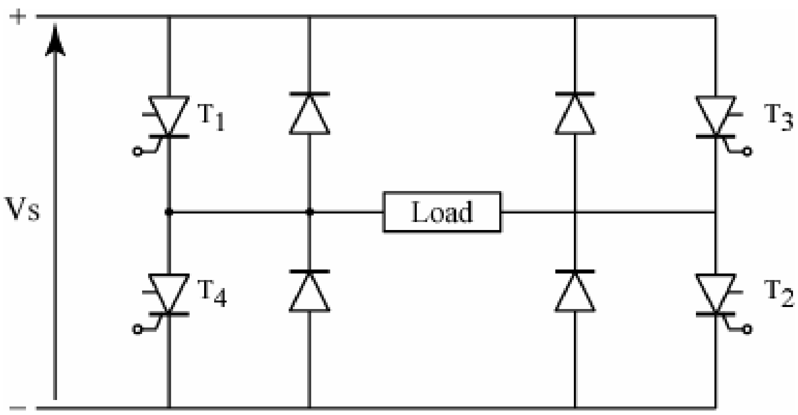
- Chopper circuits produce variable voltage DC output from constant DC supply
- Inverter circuits produce AC output from a DC supply
- AC output voltage will need to be controllable
- Frequency will sometimes need to be controlled also
- Inverters can be classified according to whether the DC supply is:
  - Constant voltage

    or
  - Constant current

- Further classification in terms of AC supply generated:
  - Single-phase

    or
  - 3-phase

- Circuit diagram:

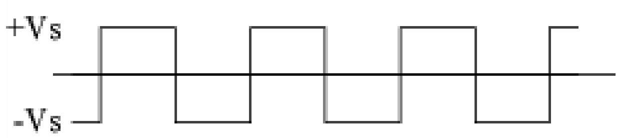


identical to circuit of class E DC chopper

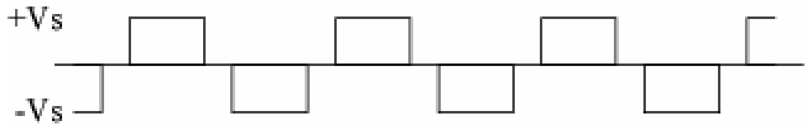
- When used as a class E DC chopper
  - Only one of four thyristors fired at any one time
  - Load assumed highly inductive for constant DC load current
  - Circuit provided chopped DC load voltage of either polarity with power flow between supply and load being in either direction
- When used as an inverter:
  - Thyristors are fired in pairs; T_1 and T_2 are fired together first and then T_3 and T_4 and so on
  - Load is assumed to be predominantly resistive
  - Effect is to produce alternating (AC) load voltage and load current where frequency can be controlled by switch device timing
- Circuit uses thyristors as switching devices with forced commutation circuits included

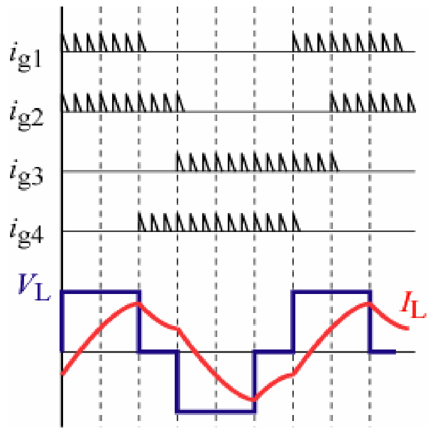
- Although load assumed predominantly resistive, it will in practice have some inductive component
- This will cause there to be small phase difference between load current and load voltage with current lagging
- Reverse diodes across thyristors accommodate phase difference between current and voltage in load; allow load current to continue flowing after each thyristor has switched off
- Important not to fire thyristor pairs T_1 - T_4 or T_2 - T_3 simultaneously as this will short-circuit supply

- If one pair of thyristors, e.g. T_3 - T_4, is fired at same instant that other pair, e.g. T_1 - T_2, is switched off, then load voltage will be square wave:



- Sometimes desirable that output voltage waveform approximates sinewave and better approximation can be achieved by delaying firing of one pair of thyristors relative to other:





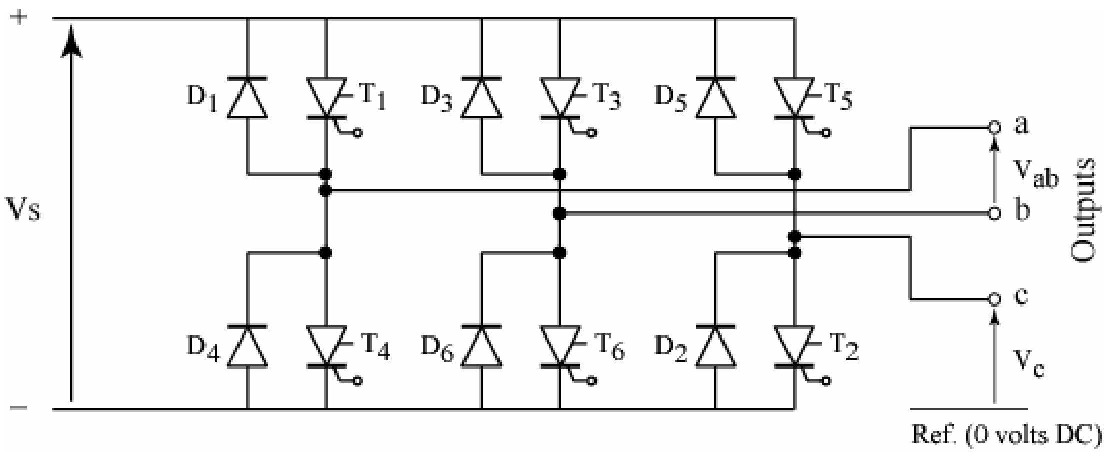
- RMS output voltage:

$$V_{RMS} = \sqrt{\frac{2}{T}\int_0^t V_L^2 dt}$$
$$= \sqrt{2}V_s\sqrt{\frac{t}{T}}$$

  - Where t is pulse width and T is switching period

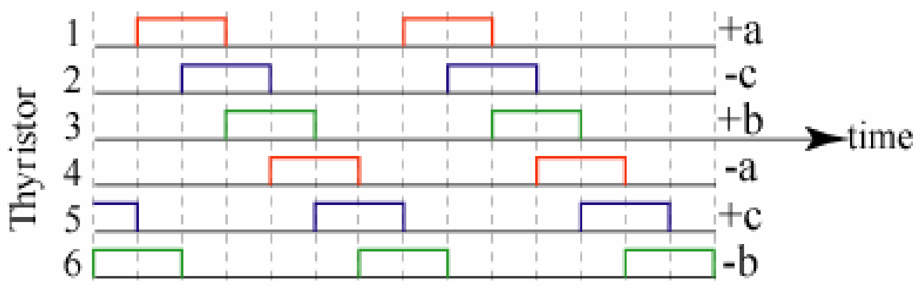
3-phase Voltage-sourced Inverter

- Combine 3 single-phase half-wave bridges:



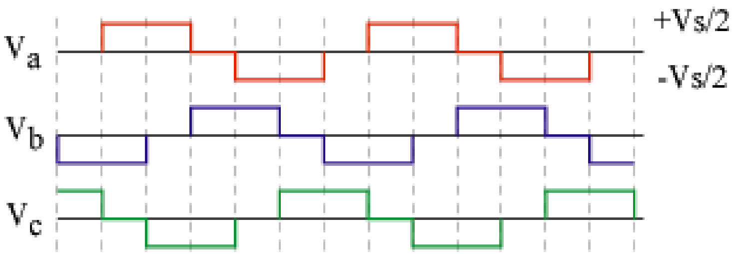
- Thyristors with commutation circuits
- Diodes included in parallel with thyristors to allow phase lag of load currents with respect to load voltages
- Assume star-connected load so load voltages are phase voltages V_a, V_b and V_c
- Typical operated with either 120º or 180º conduction angle for switching devices

- 120º conduction angle
  - Firing sequence:



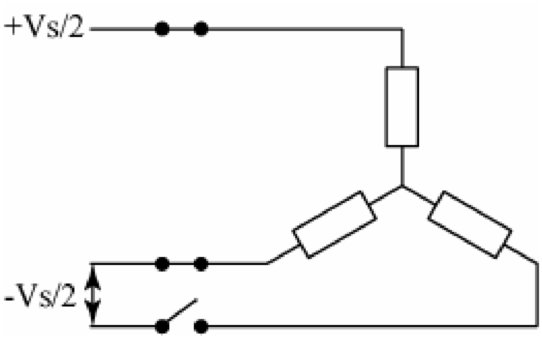
  - Conduction angle = $2\pi/3$ = 120º; firing delayed relative to previous thyristor by $2\pi/6$ = 60º

- Load voltage waveforms (phase voltages):



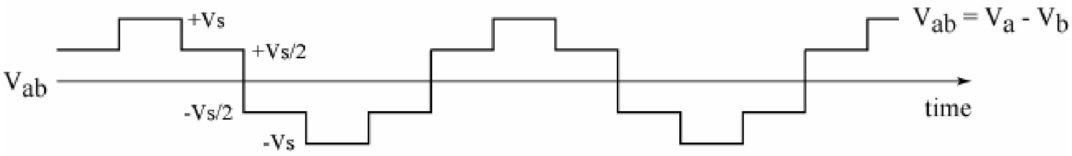
- Output phase voltage switched to +V_s/2, -V_s/2 or not connected (0 V)
- At any instant, two phase voltages are defined and one is at 0 V

- Effective instantaneous connection for star-connected load:



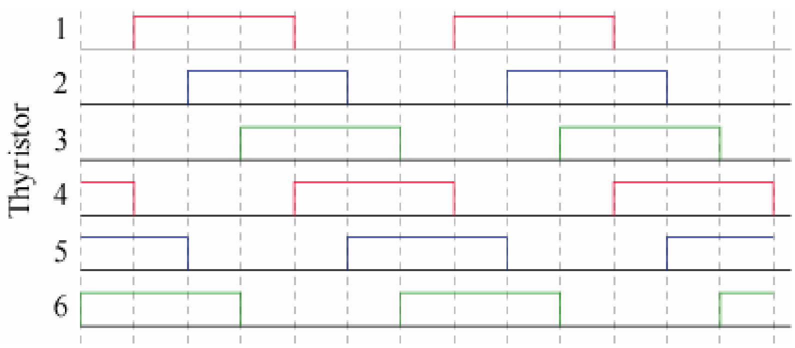
- Voltage of load star point = mean of supply voltages +V_s/2, -V_s/2 i.e. 0 V

- Each line voltage = difference a phase voltages
- E.g. $V_{ab} = V_a - V_b$:



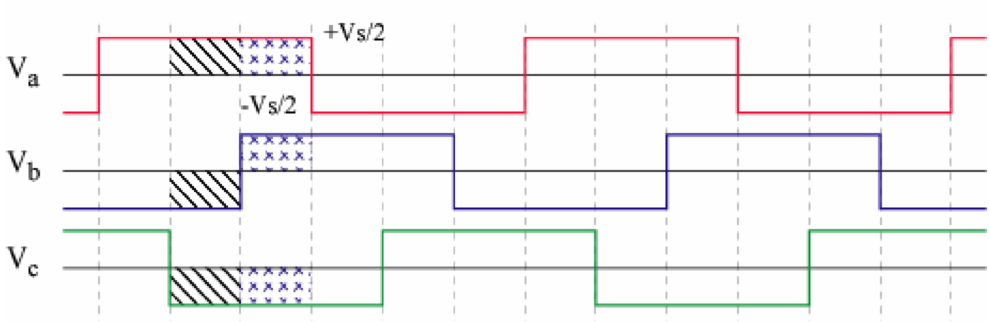
- Line voltages are of quasi-square wave form and provide better approximation to sinewave than phase voltages

- 180 conduction angle
  - Firing sequence:

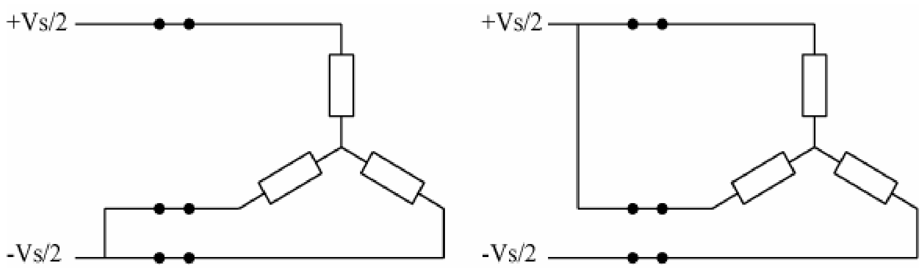


  - Conduction angle = π = 180º; firing delay relative to previous thyristor = $2\pi/6$ = 60

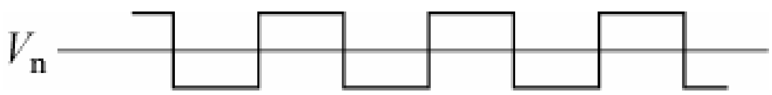
- Load voltage waveforms (phase voltages):



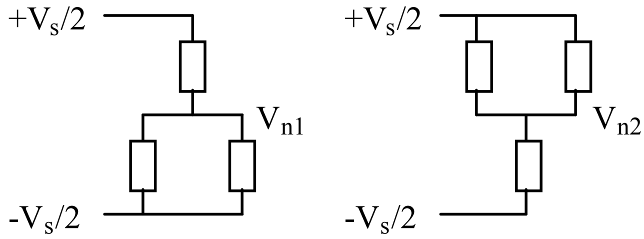
- At all times, two phase voltages at $+V_s/2$, one at $-V_s/2$, or vice versa
- Effective instantaneous connection for star-connected alternates:



- Voltage of load star point, i.e. neutral voltage, has ripple component



- Ripple voltage calculation:

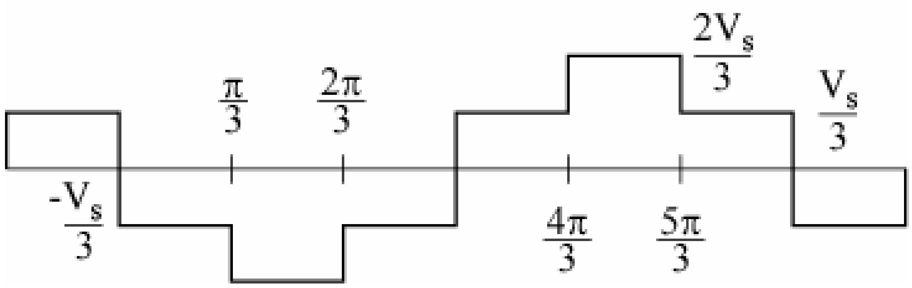


$$V_{n1} = \frac{R/2}{R/2+R}V_s + \left(-\frac{V_s}{2}\right) = \frac{V_s}{3} - \frac{V_s}{2} = -\frac{V_s}{6}$$

$$V_{n2} = \frac{R}{R/2+R}V_s + \left(-\frac{V_s}{2}\right) = \frac{2V_s}{3} - \frac{V_s}{2} = +\frac{V_s}{6}$$

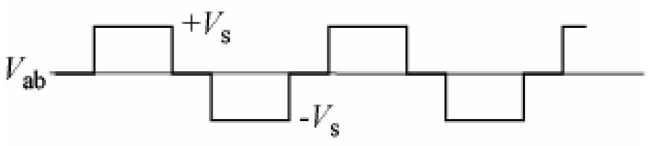
- Peak ripple = $V_s/6$ and peak-to-peak ripple = $V_s/3$

- Phase voltage V_a relative to neutral voltage:



- Phase voltages are of quasi-square wave form

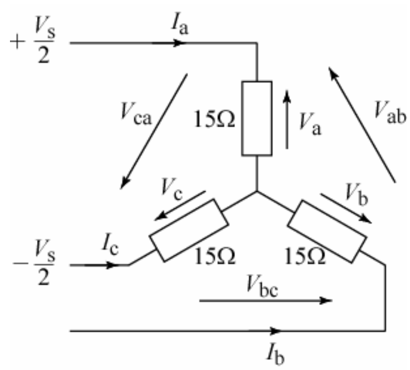
- Each line voltages = difference of phase voltages, e.g. V_{ab}:



- Self-commutating devices e.g. GTO thyristors, transistors, power MOSFETs and IGBTs removes need for forced commutation circuitry
- Also capable of operating at higher switching rates than thyristor inverters

Example On 3-phase Bridge Inverter

- A 3-phase bridge is supplied from a 600 V DC source
- The load is a star-connected resistive load of 15 Ω per phase
  - (1) Assuming 120º conduction angle, find the RMS load current, the load power and the device current ratings



- For 120º conduction angle, one of the phases of load is disconnected at all times
- Hence DC supply at all times connected to two 15 Ω resistors in series
- Load current amplitude:

$$I_L = \frac{600}{2 \times 15} = 20A$$

- RMS load current:

$$I_{L,RMS} = \left\{ \frac{1}{2\pi} \left[\int_0^{2\pi/3} 20^2 \, d\theta + \int_{\pi}^{5\pi/3} 20^2 \, d\theta \right] \right\}^{1/2}$$

$$= \left\{ \frac{20^2 + 20^2}{3} \right\}^{1/2}$$

$$= 16.33A$$

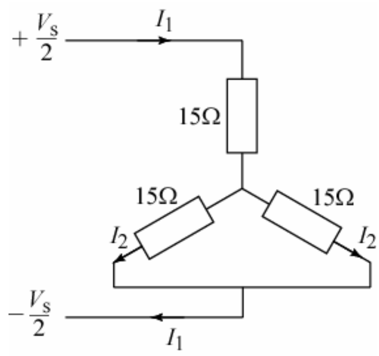
- Load power:

$$P_L = 16.33^2 \times 15 \times 3 = 12kW$$

- ∴ Thyristor RMS current:

$$I_{T,RMS} = \sqrt{\frac{20^2}{3}} = 11.5A$$

- (2) Assuming a 180º conduction angle, find the RMS load current, the load power and the device current ratings



- For 180 conduction angle, two phases of load are connected to one DC supply rail and other phase of load is connected other DC apply rail

- Hence load resistance presented to DC supply is given by:

$$R_{eq} = 15 + \frac{15}{2} = 22.5\Omega$$

- Load current amplitudes:

$$I_a = \frac{V_s}{R_{eq}} = \frac{600}{22.5} = 26.67A$$

$$I_b = I_c = \frac{V_s}{2R_{eq}} = 13.33A$$

- Phases connected in parallel for two-thirds of cycle:
- RMS load current:

$$I_{L,RMS} = \left\{ \frac{1}{2\pi} \left[\int_0^{2\pi/3} 13.33^2 \, d\theta + \int_{2\pi/3}^{4\pi/3} 26.67^2 \, d\theta + \int_{4\pi/3}^{2\pi} 13.33^2 \, d\theta \right] \right\}^{1/2}$$

$$= \left\{ \frac{2 \times 13.33^2 + 26.67^2}{3} \right\}^{1/2}$$

$$= 18.85 A$$

- Load power:

$$P_L = 18.85^2 \times 15 \times 3 = 15.99 kW$$

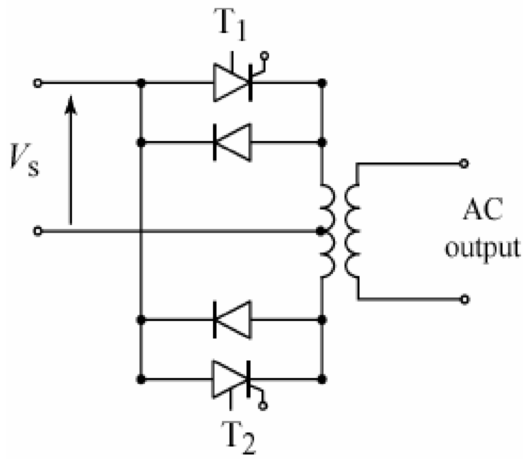
- Finally, thyristors carry current of 26.67 A for 1/6th of cycle and 13.33 A for 1/3rd of cycle

$\therefore$ RMS thyristor current:

$$I_{T,RMS} = \sqrt{\frac{1 \times 26.67^2 + 2 \times 13.33^2}{6}} = \sqrt{13.33^2} = 13.33 A$$

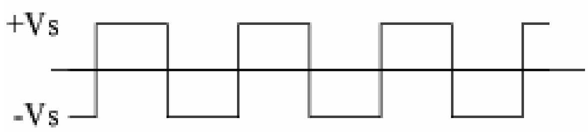
Transformer-coupled inverter

- Single-phase, transformer-coupled inverter:

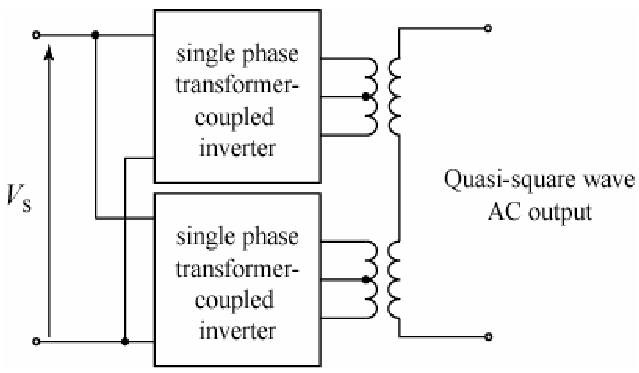


- By alternately firing and turning off thyristors T_1 and T_2, supply voltage is connected to each half of transformer primary winding in turn, producing alternating voltage in secondary winding

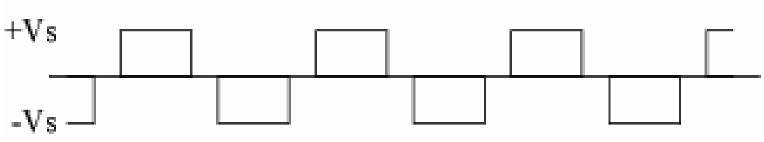
- Inverter produces square wave AC output voltage:



- Secondaries of two inverters may be effectively connected in series:

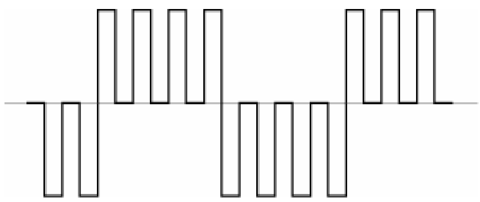


- By varying relative firing instants, quasi-square wave output is obtained:



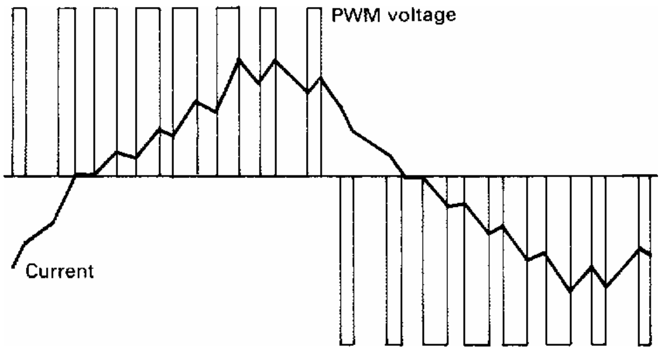
Pulse-width-modulated inverters

- Simplest form:
  - Supply voltage is switched many times within overall output voltage period in order to produce required load voltage waveform:



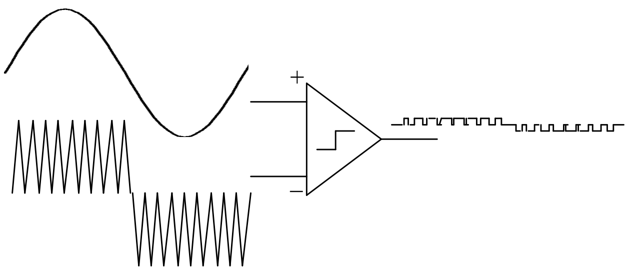
  - Mark-space ratio of pulses all equal
  - In practice, mark-space ratio of pulses varied over period in order to control load voltage amplitude within overall output voltage period

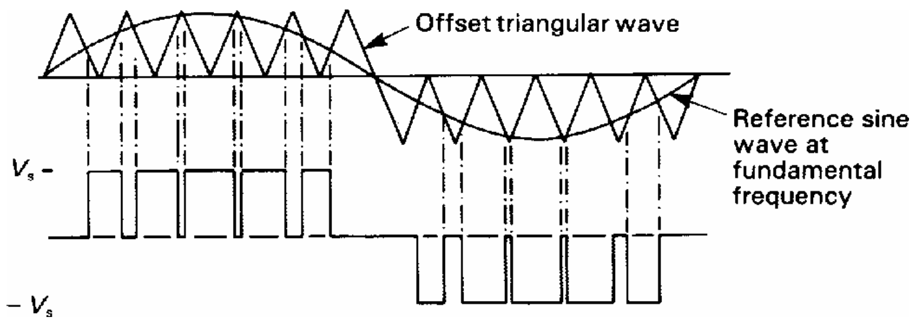
- By varying pulse widths throughout a half cycle such that $\int v dt$ value across any interval matches that of sinewave at same frequency, a near sinusoidal current can be produced in load:



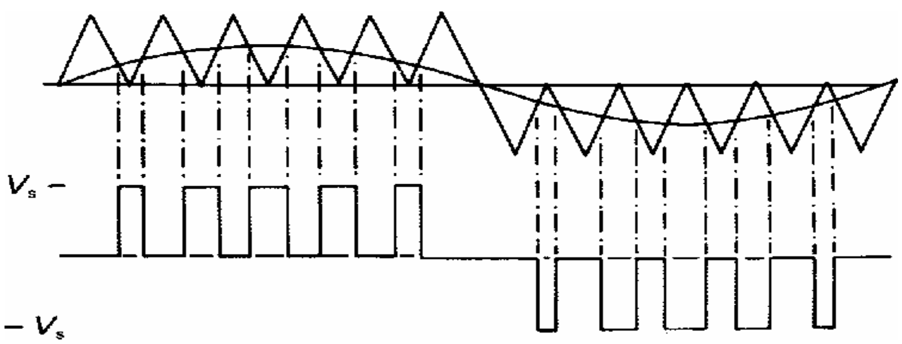
- Performance improvement obtained through reduction of harmonic content
- m chops of supply waveform in any quarter cycle of output waveform can be used to control m harmonics including fundamental
- Emphasis usually given to control or elimination of lower order harmonics as it is both easier and cheaper to filter higher frequencies as smaller capacitor and inductor values are required

- Production of PWM waveform using reference sinewave:

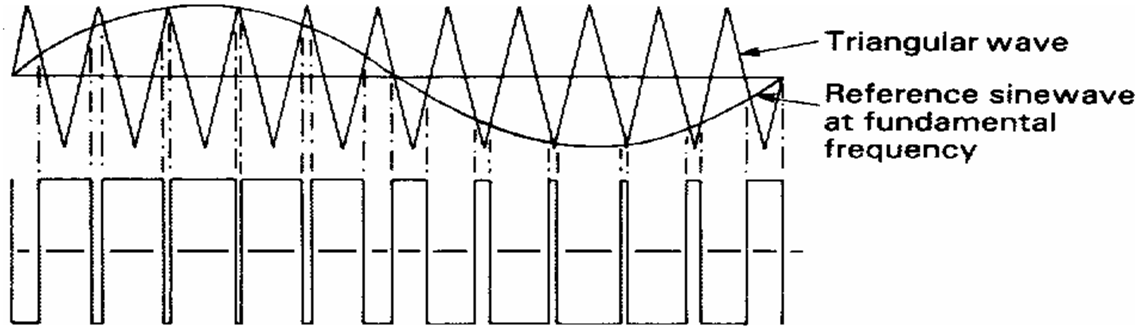




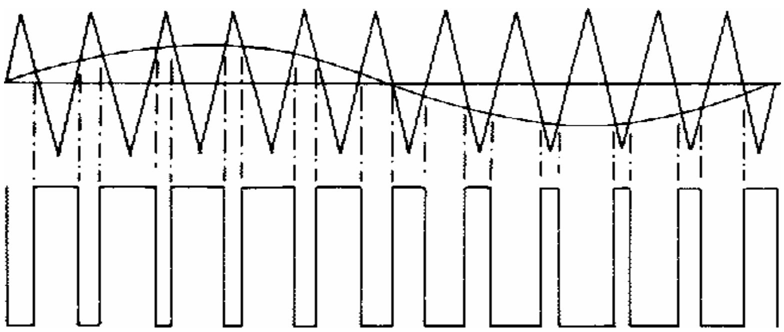
- Comparator determines instants at which waveforms cross in order to produce switching waveform
- Triangular waveform is offset positively and negatively in positive and negative half periods of sinewave
- Aim of PWM control system is to match $\int v dt$ value of output waveform with that of reference sinewave
- Reducing amplitude of reference sinewave varies widths of individual pulses to give effective lower amplitude of output wave-form



- PWM output waveform <u>tracks amplitude and frequency</u> of reference sinewave
- Switching waveform is <u>3-level</u>, positive, negative and zero
- Alternative approach giving <u>2-level</u> switching waveform uses triangular wave with <u>no offset</u>:



- When amplitude of reference sinewave is reduced, this varies widths of individual pulses to give effective lower amplitude of output waveform:

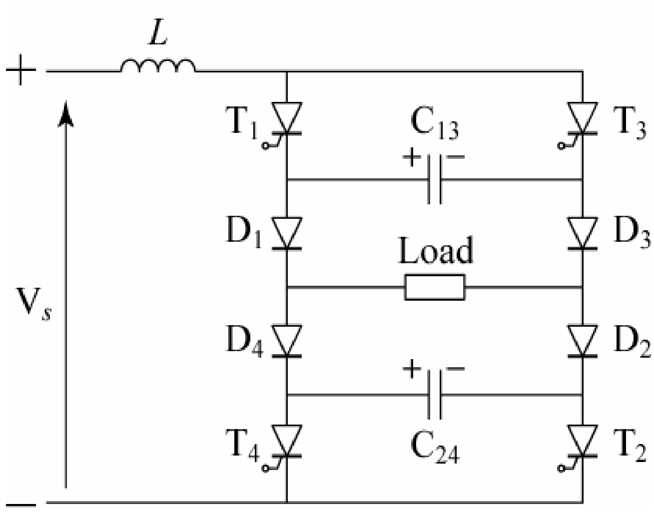


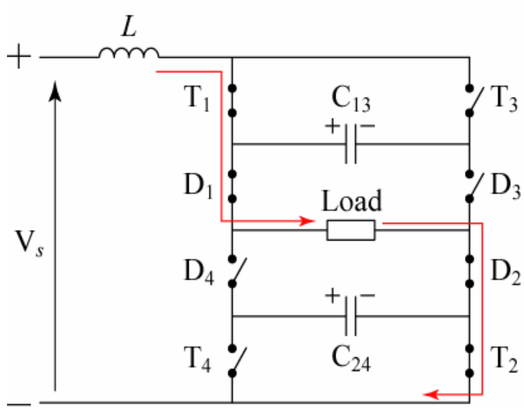
- As switching frequency is increased, switching loss becomes issue
- Implementation by ICs which essentially contain tables of pre-calculated values of switching angles covering range of output frequencies
- As computational speeds of ICs increase, it is now possible to calculate required firing angles in real time in order to optimise strategy for harmonic elimination, and control, further improving inverter performance

Current-sourced inverters

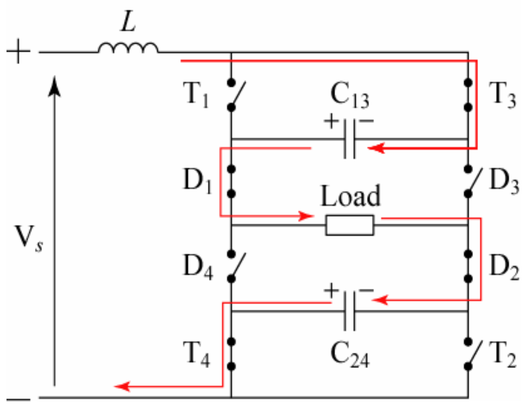
- Inductance in series with DC supply keeps supply current constant
- Series inductance provides protection against misfiring of devices and from short circuits and limits peak power rating of individual devices
- Current-sourced inverters using thyristors have advantage that commutation circuitry is simpler than in voltage-sourced inverter

- Single-phase, current-sourced bridge inverter circuit:

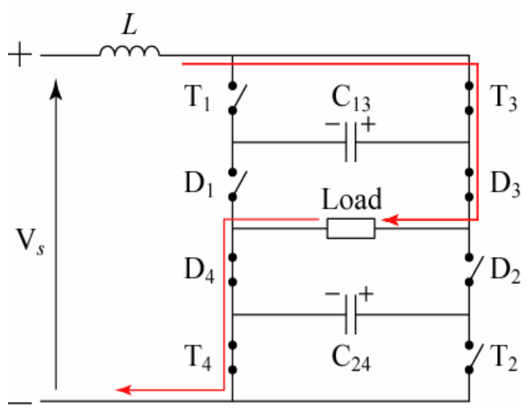




- Consider cycle of operation at point when T_1 and T_2 are conducting and commutation capacitors are charged (+ -)



- When T_3 and T_4 fired, capacitors discharge through D_1 and D_2 keeping load current in original direction; since T_1 and T_2 robbed of current and have negative voltage equal to capacitor voltage across them for > turn-off time, they are switched off



- Capacitors discharge and charge in reverse via T_3-D_1-load-D_2-T_4
- When capacitors fully charged in reverse direction, D_3 and D_4 conduct, reversing load current now carried by T_3 and T_4

Example On Single-phase Current-sourced Inverter

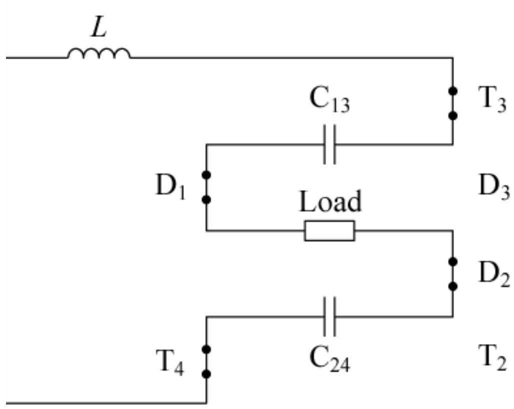
- A single-phase, current sourced inverter is being used to supply a load of 20Ω at a frequency of 24 Hz from a 160 V DC supply
- The thyristors have a turn-off time of 50 μs and the maximum permitted value of di/dt under short circuit conditions is 32 As^{-1}
- Find suitable values for the series inductance and the values of the commutation capacitors

- *Solution*
  - Maximum permitted di/dt assumes full voltage and a short circuit on the inverter side of the inductance
  - Hence

$$V_s = L\frac{di}{dt}$$

$$L = \frac{V_s}{di/dt} = \frac{160}{32} = 5H$$

- Effective connection of load immediately following firing of thyristors T_3 and T_4:



- Steady source current:

$$I_s = \frac{V_s}{R_L} = \frac{160}{20} = 8A$$

- During turn-off time of thyristor, change in current is given by:

$$\Delta I = \frac{di}{dt} t_{off} = 32 \cdot 50 \times 10^{-6} = 1.6\, mA$$

- This is negligible compared with the steady state current so we can assume constant current conditions with linear change in capacitor voltage; hence:

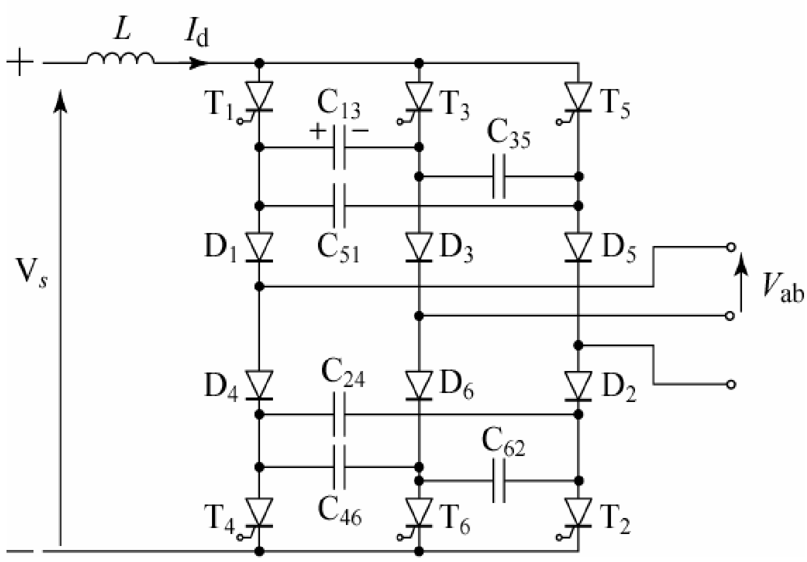
$$I_L = C\frac{dV_C}{dt} = C\frac{\Delta V_C}{\Delta t}$$

- Assuming that ΔV_c is the full supply voltage and setting $\Delta t = t_{off}$, we have:

$$C = \frac{I_L t_{off}}{V_s} = \frac{8 \cdot 50 \times 10^{-6}}{160} = 2.5 \times 10^{-6}\ F = 2.5\ \mu F$$

- Since there are two capacitors in series, they each have to have value of 5 μF

3-phase current-sourced inverter

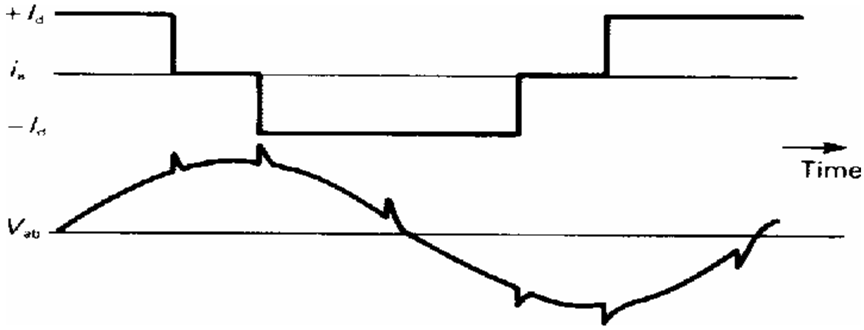


- Natural extension to 3-phase of single-phase circuit

- Thyristors conduct for 120º
- Commutation achieved by commutating capacitors C_{13} , C_{35} etc
- With T_1 and T_2 conducting, C_{13} will be charged as shown (+ –)
- When T_3 is fired as next in sequence, C_{13} initially attempts to discharge through T_1, turning it off.
- With inductive load, current will continue to flow in original direction via T_3 and D_1 until C_{13} is charged in reverse direction to V_{ab} at which point current begins to transfer to D_3

- When this transfer of current completed, commutation is complete and commutation capacitors are appropriately charged for next commutation sequence
- In fact, T_3 can be turned off by firing either T_1 or T_5, enabling any phase sequence to be achieved at output
- Diodes have additional function of isolating capacitors from any load-voltage transients during commutation

- Typical waveform of phase current and line voltage:



  - Spikes in output voltage waveform result from commutation of individual devices
  - Disadvantages of current-sourced inverters include need to filter these spikes and also their much slower dynamic response compared with voltage-sourced counterparts

Inverter performance

- Losses in inverters particularly serious because many inverters operate with very high switching rates
- Significant losses:
  - Load-dependent losses in switching devices
  - Other losses in switching devices
  - Losses in commutation circuit when forced-commutation circuits are present
  - Losses in protection circuit e.g. snubber circuits used to limit dv/dt
- Typical efficiencies:
  - 96% for quasi-square wave inverter (incl. AC/DC converter & DC link)

- 91% for PWM inverter using thyristors
- 94% for PWM inverter using transistors or GTO thyristors
- 96% for current-sourced inverters

- Typical operating (or switching) frequencies:
  - From a few hertz to 100 Hz for quasi-sine wave inverters using thyristors (higher in special applications)
  - Up to 500 Hz for inverters using bipolar transistors
  - About 100 Hz for the PWM inverter (limited by switching losses but operation at higher frequencies in a quasi-squarewave mode)
  - 5 Hz to 50 Hz for current-sourced inverters (upper limit being set by the time required for commutation)

Summary

- Have examined operation of inverters as means of producing variable-frequency, variable voltage AC source from DC supply
- Considered inverters which produce single-phase and 3-phase AC outputs
- Considered voltage-sourced and current-sourced inverters which operate from DC supplies which approximate constant voltage source or constant current source, respectively
- Introduced pulse-width-modulated inverter